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Letter to the Editor

A note on the “Exact solutions for angular motion of coaxial bodies and attitude dynamics of gyrostat-satellites”

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ABSTRACT

In this note we show by producing counterexamples that main results which appeared in the articles by Doroshin (International Journal of Non-Linear Mechanics 50, 2013) are not new solutions.

The author proposes “New analytical solutions for the angular moment components are obtained in terms of Jacobi elliptic functions. Also analytical solutions for Euler's angles are found” [1]. These solutions are given in Eqs. (2.26) and (3.2), (3.3), (3.4) [1]. Let's show that these results are not new [2–5]. The author's notations [1] retain for ease and understanding.

We begin by calculating the angular velocities. The angular velocities are obtained from the well-known kinematic Eqs. {p. 526, [2]}

$$p=\frac{\sqrt{I_2^2-L^2}}{S}(B_2+A_1)\sin\, l;\quad q=\frac{\sqrt{I_2^2-L^2}}{S}(A_2+A_1)\cos\, l$$

$$r=\frac{L-\Delta}{C_2};\quad \sigma=\frac{\Delta}{C_1}-\frac{L-\Delta}{C_2} \tag{1}$$

For ($\varepsilon=0$) Hamiltonian (1.5) [1] is given {Eq. (1.8), [2]}

$$H=\frac{I_2^2-L^2}{2}\left(\frac{\sin^2 l}{A_1+A_2}+\frac{\cos^2 l}{A_1+B_2}\right)+\frac{1}{2}\left[\frac{\Delta^2}{C_1}+\frac{(L-\Delta)^2}{C_2}\right] \tag{2}$$

It follows that (l) is a positional variable, and then (L, l) determined by Eqs.

$$\dot L=-\frac{\partial H}{\partial l};\quad \dot l=\frac{\partial H}{\partial L} \tag{3}$$

Solutions for momentum (L) of the system (3) are known {Eq. (16), [3] and Eqs. (15) and (16), [4]}

$$L=L(t)$$

and {Eqs. (49), (50) and (65), [5]}

$$s(t)=\frac{L(t)}{I_2}=\cos\,\theta(t) \tag{4}$$

And for the coordinate {[Eqs. (17), [5]]}

$$\cos\,2l=\frac{(a+b-2)s^2+4ds+4h-a-b}{(1-s^2)(b-a)} \tag{5}$$

where

$$a=\frac{C_2}{A_1+A_2};\ b=\frac{C_2}{A_1+B_2};\ s=\frac{L(t)}{I_2};\ d=\frac{\Delta}{I_2}$$

constant h is calculated for the initial conditions $t=t_0$ {[Eqs. (16), [5]]}

$$h=\frac{1}{4}[a+b+(b-a)\cos\,2l](1-s^2)+\frac{1}{2}s^2-sd$$

Solutions for the angular velocities (1) can be obtained directly by substituting the solutions (4) and (5) into (1)

$$p=\pm\frac{I_2}{A_1+A_2}\sqrt{\frac{(a-1)s^2+2ds+2h-a}{b-a}}$$

$$q=\pm\frac{I_2}{A_1+B_2}\sqrt{\frac{(b-1)s^2+2ds+2h-b}{a-b}}$$

$$r=\frac{L-\Delta}{C_2};\quad \sigma=\frac{\Delta}{C_1}-\frac{L-\Delta}{C_2} \tag{6}$$

These solutions are the simplest. Thus the Doroshin's statement «Reduction of the solutions (2.28) to the new form (2.26) by analytical transformations is problematic» {pp. 72, [1]} is unreasonable.

We now turn to the calculation of angles. Other canonical equations for Hamiltonian (2) except the (3) can be written as

$$\dot I_2=-\frac{\partial H}{\partial \phi_2}=0\Rightarrow I_2=const;\quad \dot\phi_2=\frac{\partial H}{\partial I_2}=f_{\phi_2}(L,l)$$

$$\dot\Delta=-\frac{\partial H}{\partial \delta}=0\Rightarrow \Delta=const;\quad \dot\delta=\frac{\partial H}{\partial \Delta}=f_{\Delta}(L,l) \tag{7}$$

The relative rotation angle δ and angle φ_2 can be found by calculating the quadrature (7)

$$\delta=\int f_\Delta(L(t),l(t))\,dt+C_\delta$$

$$\varphi_2=\int f_{\varphi_2}(L(t),l(t))\,dt+C_{\varphi_2}$$

The solutions for the angles θ (3.2) and (3.3) [1] obtained in the article {Eqs. (49), (50) and (65), [5]}

$$\cos\,\theta(t)=s(t)$$

and the solution for the intrinsic rotation angle also obtained {[Eqs. (17), [5]]}

$$\varphi(t)=l(t)$$

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Thus, author obtained solutions for angular velocities and for the angles which were known previously [3–5].

Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.ijnonlinmec.2013.10.007.

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