Dynamics of Large Space Debris Removal Using Tethered Space Tug

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Abstract

At present, thousands of space debris are located in Earth's orbits. It has a different size ranging from a few millimeters to tens of meters. Tethered systems are promising technology to de-orbit the space debris. The tethers have been proposed for reduction of space debris either through momentum transfer or use of electrodynamic effects. Another possible way to remove the large space debris from the critical areas of near-Earth orbit is using a tethered space tug attached to the space debris. Large space debris can strongly affect the motion of the space tug and the tether during the transportation process, which can lead to the loss of control of the tethered system. The problem of removal a large space debris from the orbit to the Earth's surface is studied. The space transportation system is composed of two bodies connected by the tether. The first body is a space debris (upper rocket stage or a large nonfunctional satellite) and the second body is a space tug. The spatial motion of the system is studied in the gravity field of the Earth under the action of the space tug thruster, aerodynamic drag and the gravitational torque. Osculating elements of the orbit are used to describe the motion of the center of mass of the system. Particular attention is given to investigate the spatial motion of the space debris relative to the tether and to the space tug. The influence of the initial conditions and the properties of the system on the motion of the system is studied.

Keywords: space debris, attitude motion, space tug, tether, thruster, deorbit

1 Introduction

The number of defunct objects (spent rocket stages, old satellites, fragments from disintegration, erosion, and collisions) in orbit around the Earth is growing very fast. The more crowded the space around the Earth becomes the more likely collisions between satellites and space debris to occur. Due to these collisions, many more dangerous pieces of debris are created. To preserve space environment for spaceflight investigations the active debris removal technologies should be considered as a high priority strategic goal of the international efforts [1-6].

Tethers look like a promising way to de-orbit old satellites [7-13]. The tethers have been proposed for reduction of space debris either through momentum transfer or use of electrodynamic effects [7, 9]. For example, in [9] satellite de-orbit modules is proposed that

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provide cost-effective, lightweight, and reliable means of removing objects from low-Earth-orbit. On the other hand, the large collision area of long tethers, combined with operational hazards and meteoroid risk may result in a large orbital exposure. Another possible ways to remove the large space debris from the critical areas of near-Earth orbit is tethered space tug ("debritor" [8]), which uses the thruster. The debritor has to perform a rendezvous maneuver, attach a tether to the space debris and de-orbit of the debris or change its orbit (Fig. 1). In [12] the tethered tug-debris system proposes to use an active upper stage rocket body with remaining fuel reserves to rendezvous with the space debris. After that a tether is connected to the debris object and burn being applied which lowers the periapsis of both objects. This scheme provides natural inherent stability of the space debris motion relative to the tether as distinct from the scheme when the space debris is pushed by the space tug [4]. In [12] the space debris and the space tug are considered as material points, but the large space debris can strongly affects the motion of the debritor and the tether during the transportation, which can lead to the loss of control of the tethered system. Proposed paper studies the influence of the large space debris to the motion of the system (tug+tether+ladge debris) during the transportation process. The dynamics and simulations of the tethered satellite systems during deployment and retrieval phase are presented in [13, 14]. Here we consider only the transportation phase (Fig. 1).

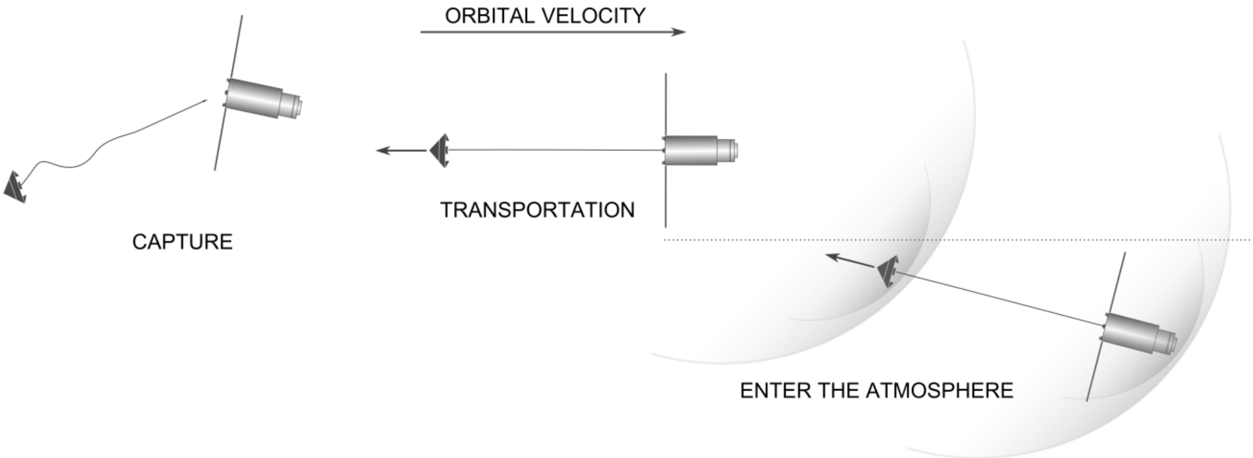


Fig. 1 Stages of the de-orbiting process

In this paper the space debris (large, passive, non-cooperative, nonfunctional spacecraft or an upper stage of the rocket) is considered as a rigid body hereinafter also referred to as a passive spacecraft. The space tag or the active spacecraft is considered as a mass point. The active spacecraft equipped with a rocket thruster and connected to the passive spacecraft by the viscous-elastic tether.

The paper consists of the four main parts. At the part 2 the motion equations of the system is provided. The equations of the relative motion of the space debris and the space tug are written in the orbital reference frame. The equations of the center of mass of the system are written using osculating elements of the orbit. At the part 3 several numerical examples is considered. The influence of the parameters of the system to its motion is examined. The correctness of the mathematical model is discussed at the part 4.

2 Motion Equation

The motion of the space debris and the space tug is considered in the rotating reference frame $Ox_oy_oz_o$ (Fig. 2). The differential equations of the centers of mass of the space debris and the space tug are the following

$$m_1\mathbf{a}_1 = \mathbf{A}^{io}(m_1\mathbf{g}_1 - \mathbf{T} + \mathbf{D}_1 + \mathbf{F}) \tag{1}$$

$$m_2\mathbf{a}_2 = \mathbf{A}^{io}\left(m_2\mathbf{g}_2 + \mathbf{T} + \mathbf{D}_2\right) \tag{2}$$

where m_1, m_2 are masses of the space tug and the space debris, $\mathbf{a}_1$, $\mathbf{a}_2$ are absolute accelerations of the space tug and the space debris, $\mathbf{A}^{io}$ is a rotation matrix that transforms the coordinates from orbital reference frame $Ox_oy_oz_o$ to the Earth centered inertial frame XYZ (all vectors in parentheses in (1) and (2) supposed to be written in the orbital frame), $\mathbf{T}$ is a tether force, $\mathbf{D}_1, \mathbf{D}_2$ are atmospheric drag forces, $\mathbf{F}$ is a space tug thruster force. The absolute accelerations of the space tug and the space debris are expressed as

$$\mathbf{a}_j = \frac{d^2}{dt^2}\left[\mathbf{A}^{io}(\mathbf{r}_o + \boldsymbol{\rho}_j)\right], \; j = 1, 2 \tag{3}$$

The positions of the space tug and the space debris relative to the center of the Earth are denoted by the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$

$$\mathbf{r}_j = \mathbf{r}_o + \boldsymbol{\rho}_j, \quad \boldsymbol{\rho}_j = [x_j, y_j, z_j]^T, \qquad j = 1, 2 \tag{4}$$

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are positions of the space tug and the space debris relative to the center of mass of the system. The vector $\mathbf{r}_o$ describes the position of the center of mass of the system, which performs an orbital motion. Vector $\mathbf{r}_o$ has the following coordinates in the frame $Ox_oy_oz_o$

$$\mathbf{r}_o = \left[\frac{p}{1 + e\cos\theta}, 0, 0\right]^T \tag{5}$$

The coordinates of the orbital velocity vector $\mathbf{V}_o$ in the frame $Ox_oy_oz_o$ is

$$\mathbf{V}_o = \begin{pmatrix} \sqrt{\mu / p}\, e \sin\theta \\ \sqrt{\mu / p}\,(1 + \cos\theta) - \Omega_o r_o \cos i \\ \Omega_o r_o \cos(\theta + \omega) \sin i \end{pmatrix}$$

where p is a focal parameter of the orbit, e is an eccentricity, θ is a true anomaly, μ is the standard gravitational parameter of the Earth, ω_o is an angular velocity of the orbital motion.

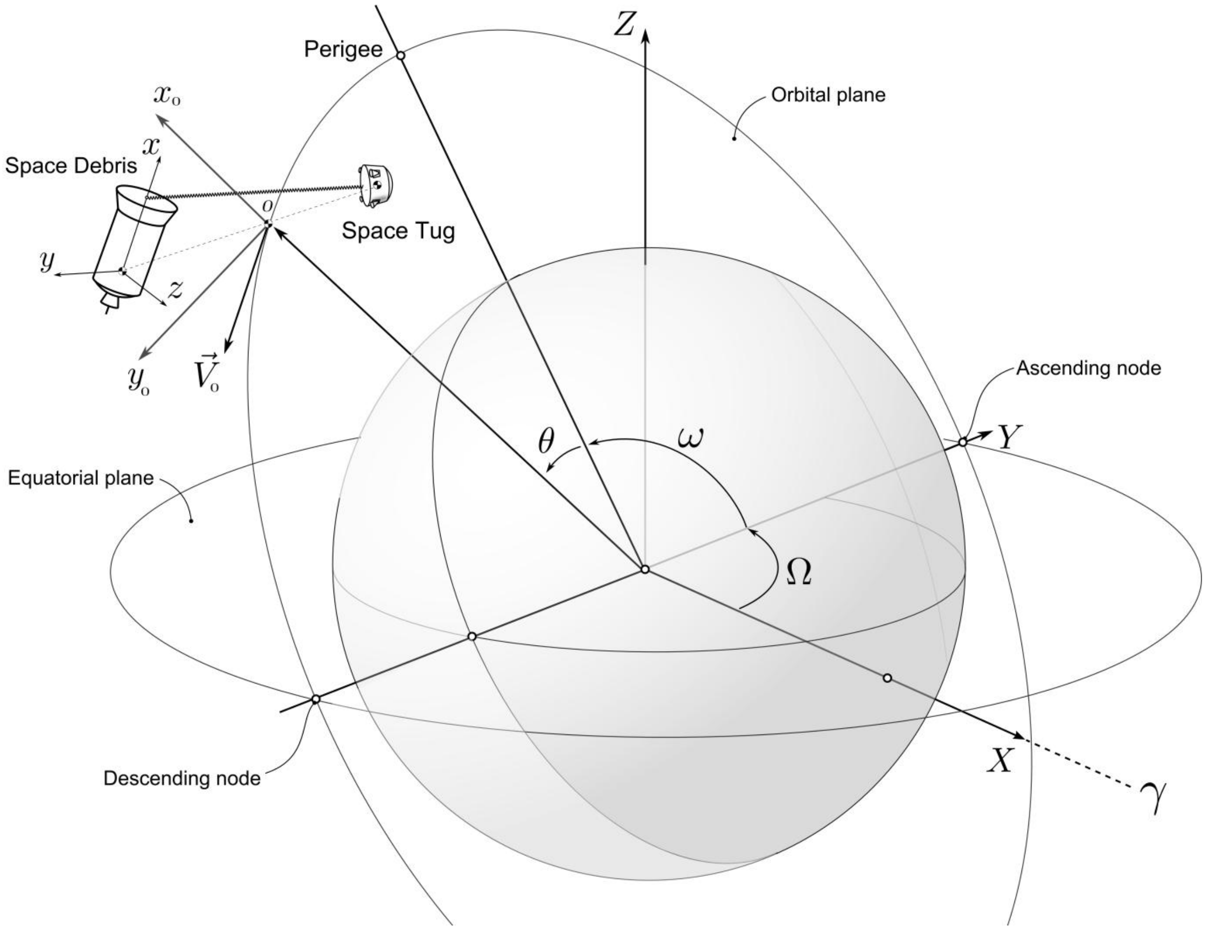


Fig. 2 The space tug and the space debris

The rotation matrix $\mathbf{A}^{io}$ is

$$\mathbf{A}^{io} = \begin{pmatrix} c_\Omega c_{\omega+\theta} - c_i s_\Omega s_{\omega+\theta} & -c_i s_\Omega c_{\omega+\theta} - c_\Omega s_{\omega+\theta} & s_i s_\Omega \\ c_i c_\Omega s_{\omega+\theta} + s_\Omega c_{\omega+\theta} & c_i c_\Omega c_{\omega+\theta} - s_\Omega s_{\omega+\theta} & -s_i c_\Omega \\ s_i s_{\omega+\theta} & s_i c_{\omega+\theta} & c_i \end{pmatrix} \tag{6}$$

where $c_\Omega = \cos\Omega, s_\Omega = \sin\Omega$, $c_i = \cos i, s_i = \sin i$, $c_{w+\theta} = \cos(w+\theta), s_{w+\theta} = \sin(w+\theta)$, i is an orbit inclination, ω is an argument of perigee. The accelerations of the gravity forces are expressed as

$$\mathbf{g}_j = -\mu \frac{\mathbf{r}_j}{\left|\mathbf{r}_j\right|^3}, \ j = 1, 2. \tag{7}$$

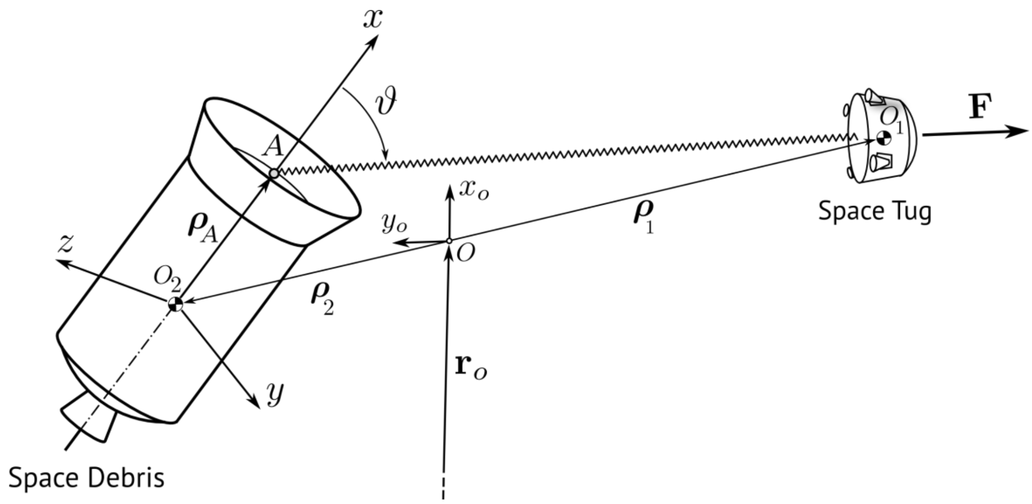


Fig. 3 The position of the space tug relative to the orbital debris

The accelerations of the space debris and the space tug due to atmospheric drag are

$$\mathbf{a}_{dj} = \frac{\mathbf{D}_j}{m_j} = -\frac{1}{2}\rho\frac{c_{dj}S_j}{m_j}V_{rj}\mathbf{V}_{rj}, \quad j = 1, 2$$

where ρ is the atmospheric density, c_{dj} is a drag coefficient, S_j is an average cross-sectional area of the spacecraft normal to its direction of travel (drag area). The drag area is directly related to the spacecraft's shape, dimensions and attitude motion. The term $BC_j = c_{dj}S_j / m_j$ is a ballistic coefficient. V_{rj} is the spacecraft's velocity relative to the atmosphere. Due to small relative velocity of the space tug and the space debris in comparison with orbital velocity we suppose that the velocity vector of the space tug and the space debris with respect to the atmosphere are equal to the velocity of the center of mass of the system relative to the atmosphere $\mathbf{V}_{ro} = \mathbf{V}_{rj}$. In the orbital reference frame vector $\mathbf{V}_{ro}$ are expressed as

$$\mathbf{V}_{ro} = \mathbf{V}_o + \begin{pmatrix} 0 \\ -\omega_{\oplus} r_o \cos i \\ \omega_{\oplus} r_o \cos(\theta + \omega)\sin i \end{pmatrix},$$

where $\omega_{\oplus}$ is an angular velocity of the Earth. We suppose that the space tug thruster force $\mathbf{F}$ of constant magnitude acts along the Oy axis of the orbital frame, i. e., in the orbital frame $Ox_o y_o z_o$: $\mathbf{F} = [0, -F, 0]^T$, $F = \text{const}$. The tether force $\mathbf{T}$ acting on the space tug and the passive spacecraft is defined as

$$\mathbf{T} = H_l(l - l_0)\left[c_T(l - l_0) + d_T\frac{dl}{dt}\right]\frac{\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 + \boldsymbol{\rho}_A}{l} \tag{8}$$

where $H_l(l-l_0)$ is a Heaviside step function, c_T is a tether stiffness, d_T is a tether damping, $\boldsymbol{\rho}_A$ is a tether attachment point vector with respect to the center of mass of the space debris, $l=\left|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2+\boldsymbol{\rho}_A\right|$ is a tether length (Fig. 3).

2.1 Orbital motion

Due to acting of the atmospheric drag and the space tug thruster force the orbital parameters change over the time. The evolution of the osculating elements of the orbit described by the following Gauss' variation equations [15] (an another form of Gauss' variation equations is presented in [16])

$$\frac{dp}{dt}=2a_y r_0\sqrt{\frac{p}{\mu}} \tag{9}$$

$$\frac{de}{dt}=\sqrt{\frac{p}{\mu}}\left[a_x\sin\theta+a_y\left(\frac{er_0}{p}+\left(1+\frac{r_0}{p}\right)\cos\theta\right)\right] \tag{10}$$

$$\frac{d\theta}{dt}=\frac{\sqrt{\mu p}}{r_0}-\frac{1}{e}\sqrt{\frac{p}{\mu}}\left(-a_x\cos\theta+a_y\left(1+\frac{r_0}{p}\right)\sin\theta\right) \tag{11}$$

$$\frac{di}{dt}=a_z\frac{r_0}{\sqrt{\mu p}}\cos(\theta+\omega) \tag{12}$$

$$\frac{d\omega}{dt}=\frac{1}{e}\sqrt{\frac{p}{\mu}}\left[-a_x\cos\theta+a_y\left(1+\frac{r_0}{p}\right)\sin\theta\right]-a_z e\frac{r_0}{p}\cot i\sin(\theta+\omega) \tag{13}$$

$$\frac{d\Omega}{dt}=a_z\frac{r_0}{\sqrt{\mu p}}\frac{\sin(\theta+\omega)}{\sin i} \tag{14}$$

where a_x, a_y, a_z are projections onto the orbital axes x_o, y_o, z_o of the accelerations of perturbative forces (e. g. the space tug thruster or the atmospheric drag), r_o is a distance from the Earth center to the center of mass of the system. For the undisturbed Keplerian orbital elements i, Ω, ω, e remain constant and the focal parameter p is the function of the true anomaly θ only.

2.2 Attitude motion of the space debris

The attitude motion of the space debris described by the Euler equations [17]

$$\mathbf{J}_2\dot{\boldsymbol{\omega}}_2+\boldsymbol{\omega}_2\times\mathbf{J}_2\boldsymbol{\omega}_2=\mathbf{M}_2 \tag{15}$$

where $\mathbf{J}_2$ is an inertia tensor of the space debris

$$\mathbf{J}_2=\operatorname{diag}(A_2,B_2,C_2) \tag{16}$$

$\boldsymbol{\omega}_2$ is an absolute angular velocity of the space debris that is

$$\boldsymbol{\omega}_2 = \boldsymbol{\Omega}_2 + \boldsymbol{\omega}_o \tag{17}$$

where $\boldsymbol{\Omega}_2$ is an angular velocity of the space debris relative to the orbital frame $Ox_oy_oz_o$, $\boldsymbol{\omega}_o$ is an angular velocity vector of the orbital frame relative to an inertial frame in the space debris reference frame O_2xyz. Vector $\boldsymbol{\omega}_o$ has the following coordinates in the reference frame O_2xyz

$$\boldsymbol{\omega}_o = (\mathbf{A}^{o2})^T \cdot \begin{pmatrix} \frac{d\Omega}{dt}\sin i \sin(\omega+\theta) + \frac{di}{dt}\cos(\omega+\theta) \\ \frac{d\Omega}{dt}\sin i \cos(\omega+\theta) - \frac{di}{dt}\sin(\omega+\theta) \\ \frac{d\Omega}{dt}\cos i + \frac{d\omega}{dt} + \frac{d\theta}{dt} \end{pmatrix} \tag{18}$$

The space debris orientation relative to the reference frame $Ox_oy_oz_o$ described by the elements of the rotation matrix $\mathbf{A}^{o2}$ that transforms coordinates from the space debris principal frame to the orbital frame $Ox_oy_oz_o$

$$\mathbf{A}^{o2} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

where a_{ij} $(i,\ j = 1,2,3)$ are functions of time.

The torque vector $\mathbf{M}_2 = \mathbf{M}_T + \mathbf{M}_g$ includes tether force torque

$$\mathbf{M}_T = \boldsymbol{\rho}_A \times \mathbf{T}\ , \tag{19}$$

and the gravitational torque [16]

$$\mathbf{M}_g = \frac{3\mu}{|\mathbf{r}_2|^3} \begin{bmatrix} (C_2 - B_2)\gamma_z\gamma_y \\ (A_2 - C_2)\gamma_z\gamma_x \\ (B_2 - A_2)\gamma_x\gamma_y \end{bmatrix}$$

where $\gamma_x, \gamma_y, \gamma_z$ are direction cosines between the axes x, y, z of the space debris and the vector $\mathbf{r}_2$

$$\gamma_x = \frac{\mathbf{r}_2 \cdot \mathbf{A}_1^{o2}}{|\mathbf{r}_2|},\ \gamma_y = \frac{\mathbf{r}_2 \cdot \mathbf{A}_2^{o2}}{|\mathbf{r}_2|},\ \gamma_z = \frac{\mathbf{r}_2 \cdot \mathbf{A}_3^{o2}}{|\mathbf{r}_2|}$$

where $\mathbf{A}_i^{o2}\ (i=1,2,3)$ are columns of the matrix $\mathbf{A}^{o2}$.There is also a torque of the drag force that we neglect. To determine space debris attitude the kinematic equations are used [17]

$$\frac{d\mathbf{A}^{o2}}{dt} = -\tilde{\mathbf{\Omega}}_2 \mathbf{A}^{o2} \tag{20}$$

where $\tilde{\mathbf{\Omega}}_2$ is an angular velocity tensor associated to the angular velocity $\mathbf{\Omega}_2$

$$\tilde{\mathbf{\Omega}}_2 = \begin{pmatrix} 0 & -\Omega_{2z} & \Omega_{2y} \\ \Omega_{2z} & 0 & -\Omega_{2x} \\ -\Omega_{2y} & \Omega_{2x} & 0 \end{pmatrix}.$$

Equations (1), (2), (9)-(15) and (20) form a closed set of equations of the spatial motion of the system (tug+tether+ladge debris).

3 Numerical simulation and analysis

3.1 Parameters of the system

Here the influence of the parameters of the system to its motion is studied, including the moments of inertia of the space debris, the length and the properties of the tether, the thruster force of the space tug and the initial conditions. Parameters of the base system are presented in Table 1.

The axisymmetric ($A_2, B_2 = C_2$) space debris is considered on the orbit with the following initial values of the parameters

$$p = 6871\, km,\ e = 0.001,\ \ \Omega = 20°,\ \theta = 60°,\ i = 60°, \omega = 90°\,. \tag{21}$$

The space debris has initial angular velocity around its C_2x axis Ω_{2x}=0.05 rad/s.

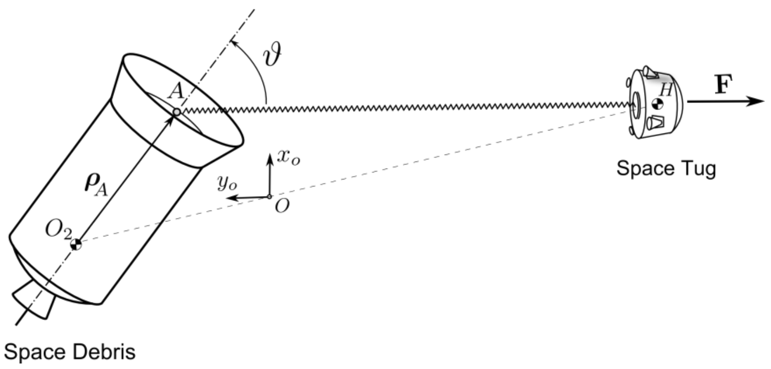
Table 1 Parameters of the base system

Parameter	Value	Parameter	Value
A_2	3000 kg·m^2	$B_2 = C_2$	15000 kg·m^2
F	20 N 5 N (case 5)	m_2	3000 kg
m_1	500 kg	c_T	60 GPa
l_0	30 m 100 m (case 4)	d_T	0 16 N·s/m (case 3)
c_{d1}	2	c_{d2}	2
S_1	1 m^2	S_2	18 m^2

In the cases 1, 2, 3 and 4 the motion of the system with initially tensioned and initially slackened tether is considered. The influence of the tether length and the tether damping to the attitude motion of the space debris is investigated. The tethers with the length $l_0 = 30\,m$, $l_0 = 100\,m$ and with the damping coefficients $d_T = 10\,N \cdot s / m$ and $d_T = 0$ are considered. Initial positions of the space tug and the space debris for the cases 1-4 are shown at Fig 4a.

In the case 5 and 6 the influence on the system under of the space tag thruster force is examined. Initial positions of the space tug and the space debris are shown at Fig 4b for these cases and further. In the case 7 full simulation of the descent process from the orbit (21) to the edge of the atmosphere $h \approx 100\ km$ is presented.

a) Cases 1-4



b) Cases 5-9

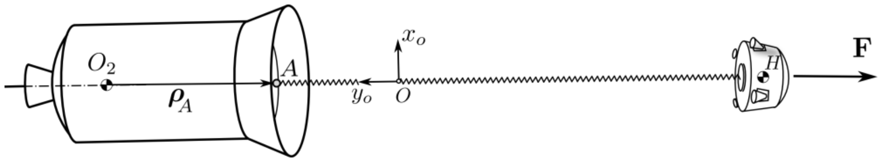


Fig. 4 The initial positions and the orientations of the space tug and the space debris

3.2 Case1

Let us consider the motion of the system with initially tensioned tether. The initial angle between the axis O_2x and the force vector $\mathbf{F}$ is $\pi / 6$ rad. The time history of angle ϑ and the tether elongation $l - l_0$ are given at Fig. 5. Fig. 5 shows that two modes of oscillation occur. A high frequency longitudinal oscillation of the tether (Fig. 5b) and low frequency precess motion the space debris relative to the tether due to the initial angular momentum of the space debris.

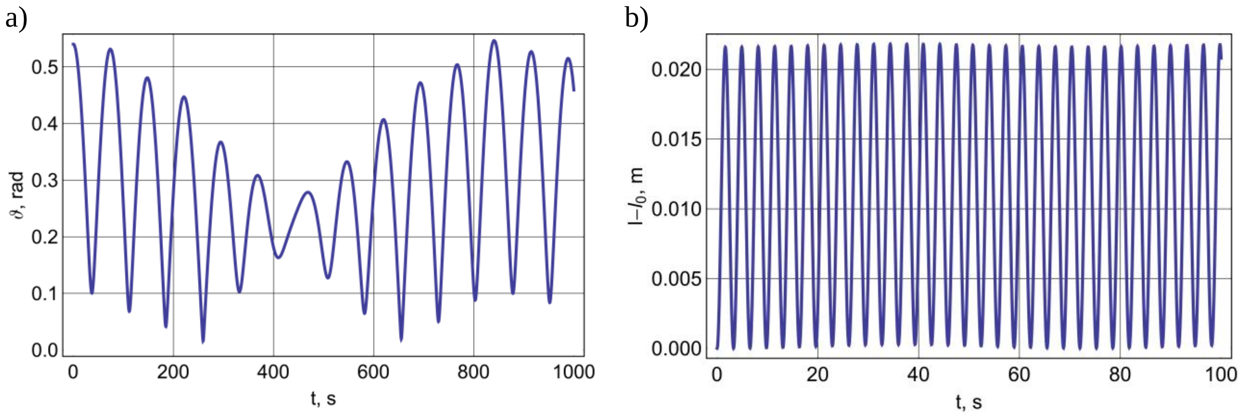


Fig. 5 Time history of the angle ϑ and the tether elongation for the case 1

3.3 Case 2

Next we consider the motion of the system with initially slackened tether. At $t = 0$ $l(0) = 27\,m < l_0$. At Fig. 6a the time history of the angle ϑ is shown. At Fig. 6b the time history of the tether elongation is presented. The amplitude of the oscillation of the angle ϑ is higher than in the case 1. We can expect greater tension of the tether in this case. It's obvious that the high oscillation of the angle ϑ during de-orbiting of the space debris should be avoided. It can lead to the tether break or tether tangles.

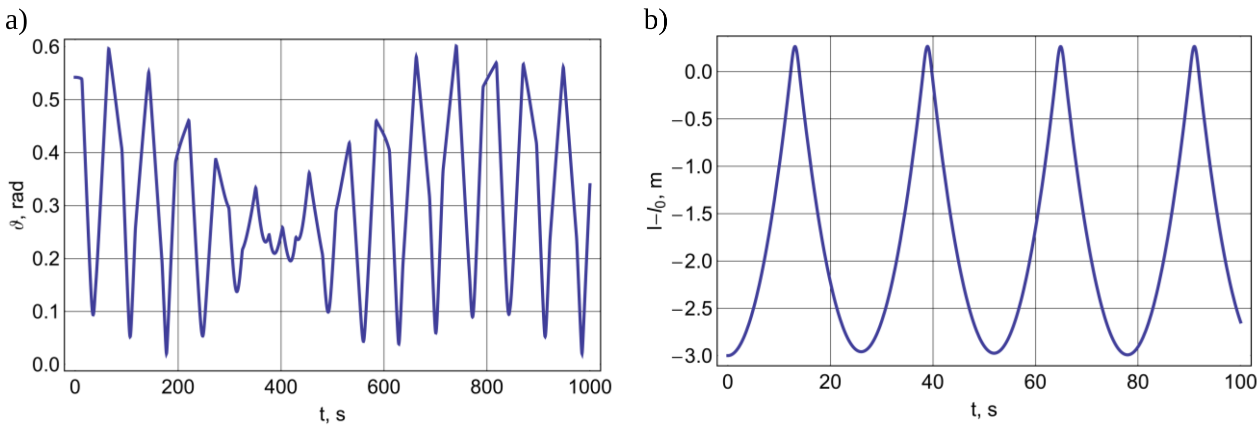


Fig. 6 Time history of the angle ϑ and the tether elongation for the case 2

3.4 Case 3

At Fig. 7 the time history of the angle ϑ is shown for the initially slackened tether ($l(0) = 27\,m < l_0$) with damping coefficient $d_T = 16\ N \cdot s / m$. The amplitude of the oscillation of the angle ϑ is a smaller than in the case 2, but effect of the tether damping on oscillation of the space debris relative to the tether is insignificant.

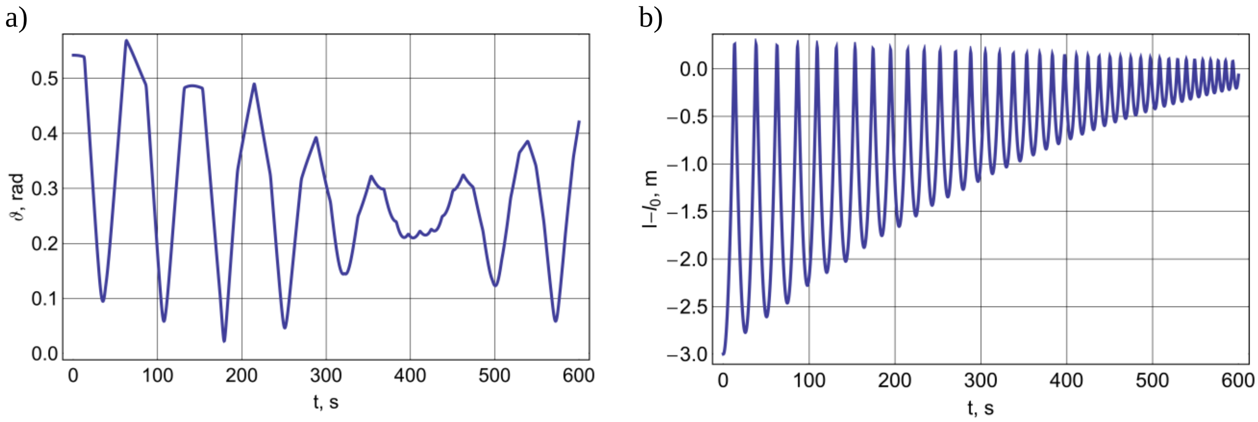


Fig. 7 Time history of the angle ϑ and the tether elongation for the case 3

3.5 Case 4

Let us consider the attitude motion of the space debris with the longer tether. At Fig. 8 the time history of the angle ϑ and the tether elongation is shown for the system with $l_0 = 100\ m$ .We note that the amplitude of the angle ϑ doesn't differ sufficiently from the case 1.

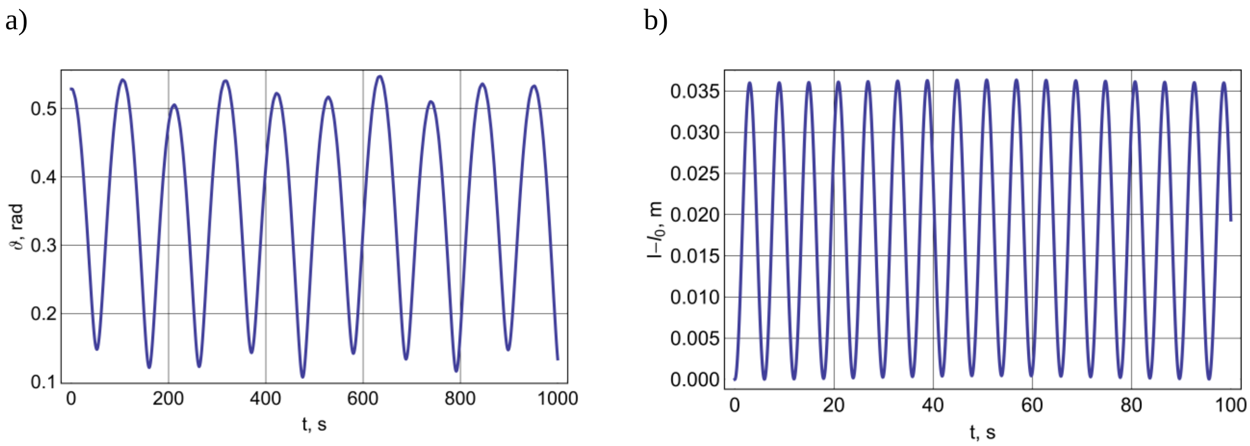


Fig. 8 Time history of the angle ϑ and the tether elongation for the case 4 (l_0 = 100 *m*)

3.6 Case 5

Here the behavior of the system is investigated when the space tug thruster force has small value $F = 2\ N$ and the initial value of angle ϑ is equal to 0. Fig. 9a shows the time history of angle ϑ. Angle ϑ is increased due to the orbital motion of the space debris and its initial angular momentum.

3.7 Case 6

The Fig. 9b shows how the angle ϑ changes when the space tug force is ten times greater than in the previous case ($F = 20\ N$). We note the ten times smaller amplitude oscillation of the angle ϑ compared to case 5. The thruster force must be sufficed to retain small angle ϑ.

a) $F = 2\ N$ b) $F = 20\ N$

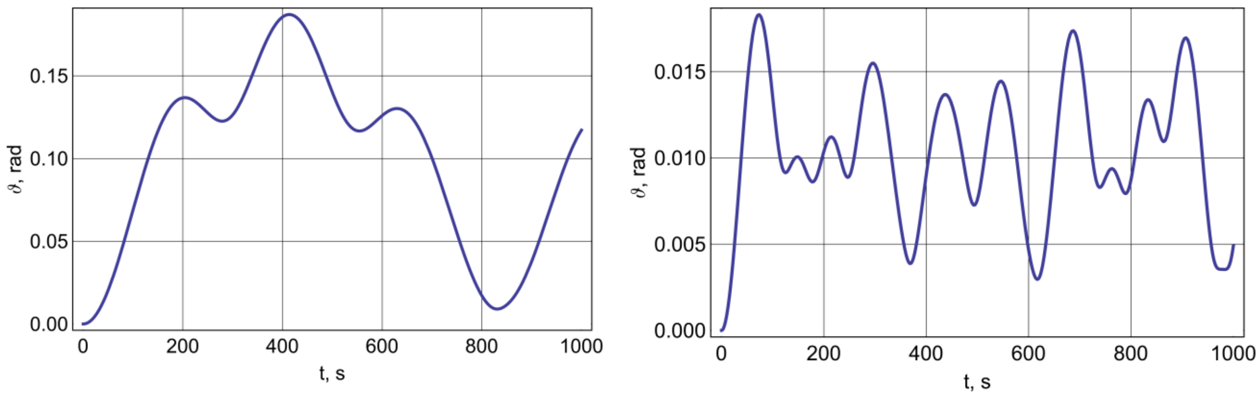


Fig. 9 Time history of the angle ϑ for the cases 5 and 6

3.8 Case 7

At last, let us consider the descent process of the space debris from the near-circular orbit with $h \approx 500\,km$ to the edge of the Earth's atmosphere $h = 100\,km$. The results of the simulation are presented at Fig. 10. Fig. 10a shows the time history of the angle ϑ and the evolution of the orbit height of the system. We note that the height of the space debris falls below 100 km after the 7 turns around the Earth. The life time of the space debris on the initial orbit at least six years [18]. The space tug de-orbit the space debris in 11-12 hours. Note that after 11 hours the tether slacked and two spacecraft approach each other. The space tug can come into collision with the connected space debris. This collision can increase uncertainty of the initial conditions of the space debris at the beginning of the atmospheric stage of the descent process.

a) b)

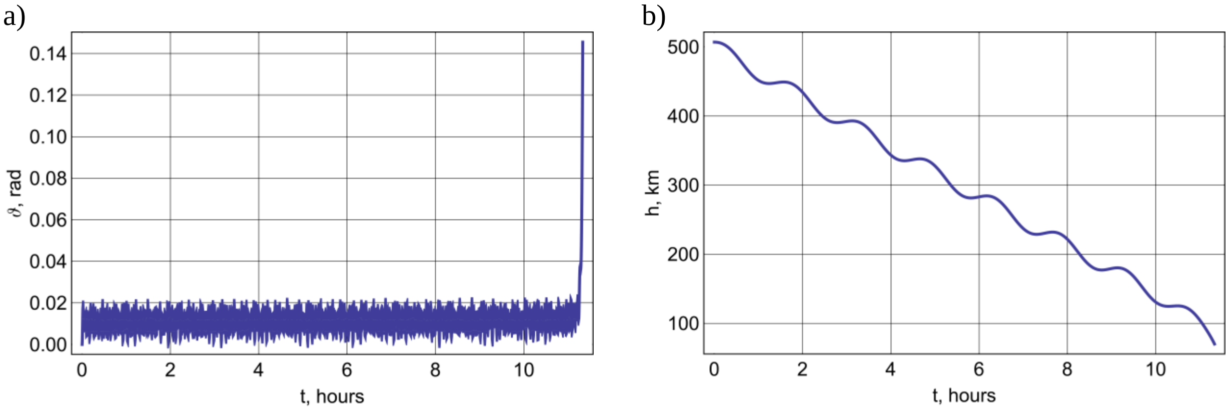


Fig. 10 Time history of the angle ϑ and the tether elongation for the case 8

Let us provide detail analysis of the motion of the system at the low altitude (100-120 km) where the atmospheric drag is essential. The atmospheric drag is the main non-gravitational force that acts on a satellite in LEO. Drag is part of the total aerodynamic force that acts on a body moving through an atmosphere. It acts in the direction opposite of the velocity. The large space debris with large cross section area can slow down by the atmospheric drag much stronger then the space tug. In this case tether slacks and the space debris can collide with the space tug. To avoid tether slackness the space tug thrust force F should be such that (Fig. 11a)

$$\frac{D_1 + F}{m_1} - \frac{D_2}{m_2} > 0 \tag{22}$$

or

$$F > \frac{m_1}{m_2} F_2 - F_1 = m_1 \cdot q(h) \cdot \left(\frac{1}{BC_2} - \frac{1}{BC_1} \right) = m_1 \cdot q(h) \cdot BC_1^* \tag{23}$$

where $q = 0.5\rho(h)V_o^2$ is a dynamic pressure BC_1 is a ballistic coefficient of the space tug, BC_2 is a ballistic coefficient of the space debris. The minimal space tug force that ensure the tension of the tether is

$$F_{\min} = q(h) m_1 \left[\frac{1}{BC_2} - \frac{1}{BC_1} \right] \tag{24}$$

At Fig.11b the minimal force of the space tug as a function of height is shown for two types of the space debris. The atmosphere density approximated by the function

$$\rho(h) = \rho_0 e^{-h/7000}, \ \rho_0 = 1.1\, kg / m^3$$

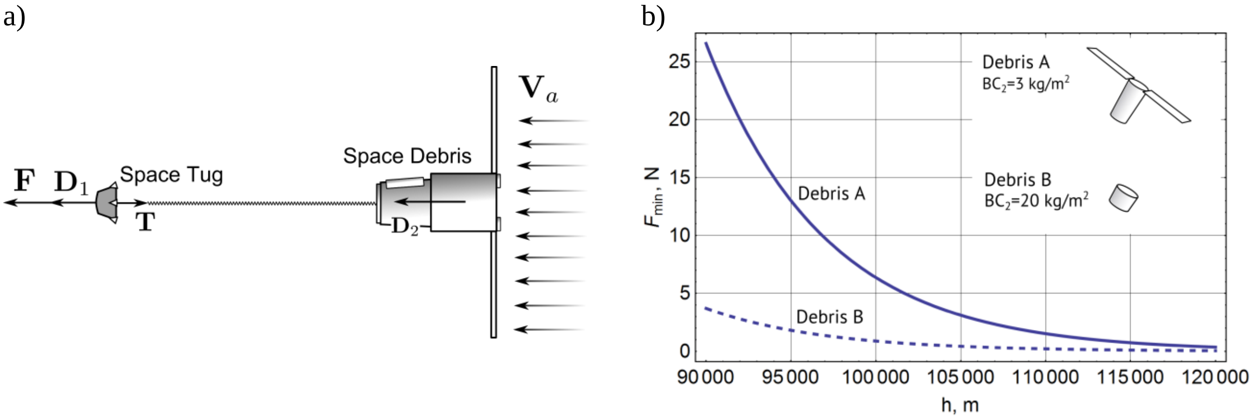


Fig. 11 Minimal space tug force for two types of the space debris

(the ballistic coefficient of the space tug BC_1=250 kg/m^2)

For the particular system parameters the minimal height of the safe transportation exists below which the tether can slack. This height depends on the maximum thruster force of the space tug and on the ballistic coefficients of the space tug and the space debris. Below this height the space tug can come into collision with the connected space debris. The collision can increases uncertainty of the initial conditions of the space debris at the beginning of the atmospheric stage of descent process. The space tug has to perform reorient maneuver before re-entering Earth's atmosphere to prevent the collision with the space debris.

4 Model correctness

The motion of the space debris and the space tug is considered relative to the center of mass of the system. The position of the space tug and the space debris relative to the center of mass of the system described by the vectors $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ respectively. The vectors $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are obtained from the independent differential equations (1) and (2). For the center of mass vector of the system $\boldsymbol{\rho}_C$ takes place the following expression

$$\boldsymbol{\rho}_C(m_1 + m_2) = m_1\boldsymbol{\rho}_1 + m_2\boldsymbol{\rho}_2 = 0 \tag{25}$$

For the correct mathematical model $\boldsymbol{\rho}_C$ should be equal to zero vector or close to the zero vector due to errors the numerical integration process. During numerical simulation the error of the position of the center mass is tested. For all considered cases norm of the vector $\boldsymbol{\rho}_c$ is less than 0.001 meters.

Conclusion

The influence of the parameters of the system to its motion, including the moments of inertia of the space debris, the length and the properties of the tether, the thruster force of the space tug and the initial conditions of the motion is studied. The safe transportation process is possible when the space tug force vector coincides with the direction of the tether and the tether is always tensioned. Tether damping device slightly reduces the amplitude oscillations of the space debris. The space tug has to keep sufficient level of the thruster force to eliminate the high amplitude oscillations of the space debris relative to the tether. There is the minimal height of the safe transportation below which the space tug can come into collision with the connected space debris.

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