

Behaviour of Tethered Debris With Flexible Appendages

Vladimir S. Aslanov*, Vadim V. Yudintsev*

34, Moskovskoye shosse, Samara, 443086, Russia

Abstract

Active exploration of the space leads to growth a near-Earth space pollution. The frequency of the registered collisions of space debris with functional satellites highly increased last ten years. As a rule a large space debris can be observed from the Earth and catalogued, then it is possible to avoid collision with the active spacecraft. However every large debris is a potential source of a numerous small debris particles. To reduce debris population in the near Earth space the large debris should be removed from working orbits. The active debris removal technique is considered that intend to use a tethered orbital transfer vehicle, or a space tug attached by a tether to the space debris. This paper focuses on the dynamics of the space debris with flexible appendages. Mathematical model of the system is derived using the Lagrange formalism. Several numerical examples are presented to illustrate the mutual influence of the oscillations of flexible appendages and the oscillations of a tether. It is shown that flexible appendages can have a significant influence on

*Corresponding author.

Email addresses: aslanov_vs@mail.ru (Vladimir S. Aslanov),
yudintsev@classmech.ru (Vadim V. Yudintsev)

URL: <http://aslanov.ssau.ru> (Vladimir S. Aslanov)

the attitude motion of the space debris and to the safety of the transportation process.

Keywords: active debris removal, tether, space tug, relative motion, flexible appendages, vibrations

1. Introduction

The first Russian Sputnik satellite that is launched in 1957, stayed in orbit for three months only. Last half century more than six thousand spacecraft was launched to the Earth orbits and many of them remain in orbit. There are more than 15000 large objects on the orbits around the Earth. Only 7% of these are active spacecraft, 17% are nonfunctional spacecraft and 13% are orbital stages of the rockets [1]. All these objects are tracked and an active spacecraft or a space station can avoid collision with such objects. Collisions of the large space debris with other debris can significantly increase numbers of the small debris on the Earth orbit. The Fengyun 1C anti-satellite test [2] and the Cosmos-Iridium collision [3] created over 5000 small objects [4]. The debris cascade effect described by Kessler [5] has begun to occur. Several orbits can be dangerous for the new missions therefore large debris should be removed. Removal of five or more large debris per year can reduce the debris population [6].

Last years several active debris removal methods were developed [7–14]. At fig.1 one of the possible classification of the active debris removal is shown. There are three types of the connection between a space tug and a space debris: flexible connection, rigid connection and distant interaction. The last case applies to the techniques based on the idea of thrusting a space

debris by irradiating it with an ion beam [15]. Rigid connection between a space tug and a debris can be realized by robot arm. The flexible connection can be provided by a tether attached to the space debris.

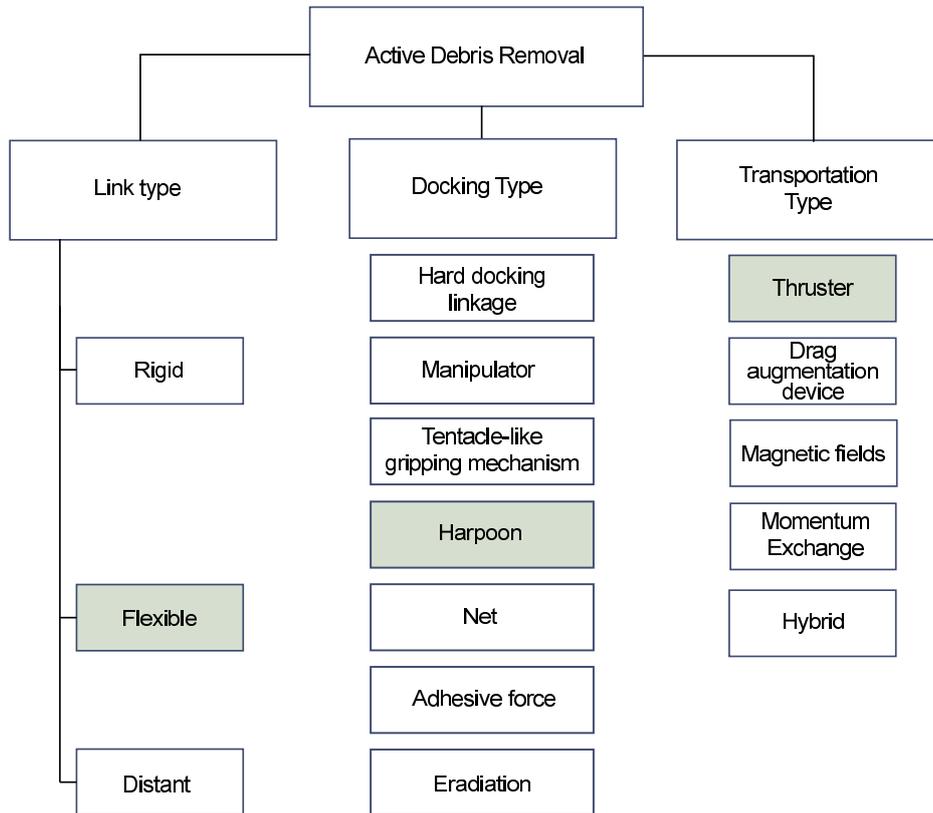


Figure 1: Active debris removal classification

In our opinion, the tethered transportation with the pulling space tug has the following advantages over the rigidly connected space tug and space debris:

- Lower requirements for the tug's control system system, because of natural stability of the pull scheme [1];

- Transportation is safe for the space tug: in the case of breaking of connection with the debris the transportation attempt can be repeated.

The active debris removal mission can be divided into several stages specified by the motion pattern of the space tug relative to the space debris [7].

1. Placing the space tug into orbit.
2. Far-range rendezvous between the space tug and the debris.
3. Rendez-vous phase.
4. Mechanical interfacing (docking, grapping, etc.).
5. De-tumbling and orientation of the space debris.
6. Thruster-burn phase.
7. De-orbitation (post-burn) phase.
8. Enter to the atmosphere.

Each stage requires a different mathematical model. Note that mathematical models for the steps 1 to 3 and 7, 8 are well known. To analyse these steps do not require the creation of any new models in addition to the existing models of the orbital motion of the spacecraft.

Post-burn phase are considered in [16–18] where discussed thruster input shaping techniques to reduce the post-burn relative motion between space tug and space debris. The motion of the tug-tether-debris system as a material point, assuming stationarity of the relative motion of the tug and the debris for the stage 7 can be described by differential equations in the osculating parameters [19]. The atmospheric entry can be analysed using the mathematical models presented in [20], [21].

The choice of the active debris removal technique depends on the properties of the space debris. Reference [1] notes that there are two types of the

space debris: spacecraft or orbital stages. Orbital stages are more “comfortable” for the deorbit, because they don’t have large appendages (solar panels, antennas).

The removal of passive spacecraft with flexible appendages is more complex problem. The possibility of a vibration of flexible appendages should be considered that may leads to the destruction of the spacecraft and the emergence of an even greater number of small fragments.

In this paper we draw attention to the stage 6 of the active removal of a space debris with flexible appendages. The aim of the present work is to derive a mathematical model to perform a research of the influence of flexible appendages of space debris (passive spacecraft) to the initial phase of the deorbit process. We consider the simple impulsive burn of the tug’s thrust. As noted above, input shaping techniques can be used to reduce the post-burn relative motion between space tug and space debris [17]. An alternative solution to remove collision potential is the use of post-burn manoeuvre of the space tug after detaching the tether to establish a safe relative orbit of the tug. We suppose that a thrust force acting on the space tug provides torque much greater magnitude compared to the gravitational torque. [22] considers the dynamics of large debris with flexible appendages, but the flexible appendages are modeled as rigid bodies connected to the space debris with viscoelastic joints. In our paper flexible appendages are considered as in-plane bending homogeneous beams.

Note 1

The paper has two main sections. In the next section the motion equations of the space debris with flexible appendages are formulated. In the later section these equations are used to explore the motion of the system in

several cases in terms of numerical simulation.

This paper continues our research in [23] and [24] where considered the attitude motion of the space debris as a rigid body without flexible appendages in a space without gravity [24] and in a central gravitational field [23].

2. Equations of the space debris with flexible appendages

2.1. Lagrange formalism and generalized coordinates

The equations of motion of the debris relative to the space tug can be written using Lagrange or Newton-Euler formalism. The obvious advantages of this method are the minimal set of generalized coordinates describing the configuration of the system, the possibility of conducting analytical studies the equations – their linearization and simplification. Also, it is simple to incorporate the flexing dynamics using Lagrange formalism.

To analyze the safety of orbital transportation process the relative motion of the debris and the space tug should be considered. From this point of view the motion equation should be written in the orbital coordinate system with origin in the center of mass of the system (tug + tether + debris).

The non-inertial effects are systematically neglected as far as a short period of time of the de-orbit stage is studied (from the space tug's thruster burn). The motion of the system's center of mass can be described using equations for osculating orbital elements elements [23].

The configuration of the considered system is described by the following set of generalized coordinates $s = (x, y, z, \psi, \vartheta, q_{ij}, l, \alpha_1, \alpha_2, \alpha_3)$. Coordinates x, y, z determine position of the center of the debris (passive spacecraft) relative to the orbital frame, angles ψ, ϑ describe orientation of the tether relative

to the space debris, l denotes the tether length and coordinates, q_{ij} is a subset of the modal coordinates of the i flexible appendage. The orientation of the debris is parameterized with Bryant angles $\alpha_1, \alpha_2, \alpha_3$ (x-y-z rotation sequence) [25] that define orientation of the debris relative to the orbital frame. This angle set has singularity at $\alpha_2 = \pi/2$, but the motion of the debris near the angle $\alpha_2 = \pi/2$ unpredictable due to the possibility of the entanglement of the tether, therefore it is supposed that $\alpha_2 < \pi/2$ (Fig. 2). Lagrange equations has the following form

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{s}_k} - \frac{\partial K}{\partial s_k} = Q_k \quad (1)$$

where K is a kinetic energy of the system, Q_k is a generalized force correspond to the generalized coordinate s_k . The kinetic energy of the considered system consist of two terms: kinetic energy of the rigid bodies and the kinetic energy of the flexible appendages. Before presenting the expression for the kinetic energy let us consider the kinematics of the system.

2.2. System's kinematic

The velocities of the space tug \mathbf{v}_1 and the space debris \mathbf{v}_2 in $Ox_0y_0z_0$ frame are

$$\mathbf{v}_2 = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T, \quad \mathbf{v}_1 = \frac{d\mathbf{r}_1}{dt}$$

where

$$\mathbf{r}_1 = \mathbf{r}_2 + \mathbf{M}_\alpha(\boldsymbol{\rho}_A + \mathbf{n}_T l) = \mathbf{r}_2 + \mathbf{M}_\alpha(\boldsymbol{\rho}_T + \mathbf{l})$$

where $\boldsymbol{\rho}_A$ is a vector of the tether attachment point A . \mathbf{M}_α is a rotation matrix that transforms coordinates from the debris frame $C_2x_2y_2z_2$ to the

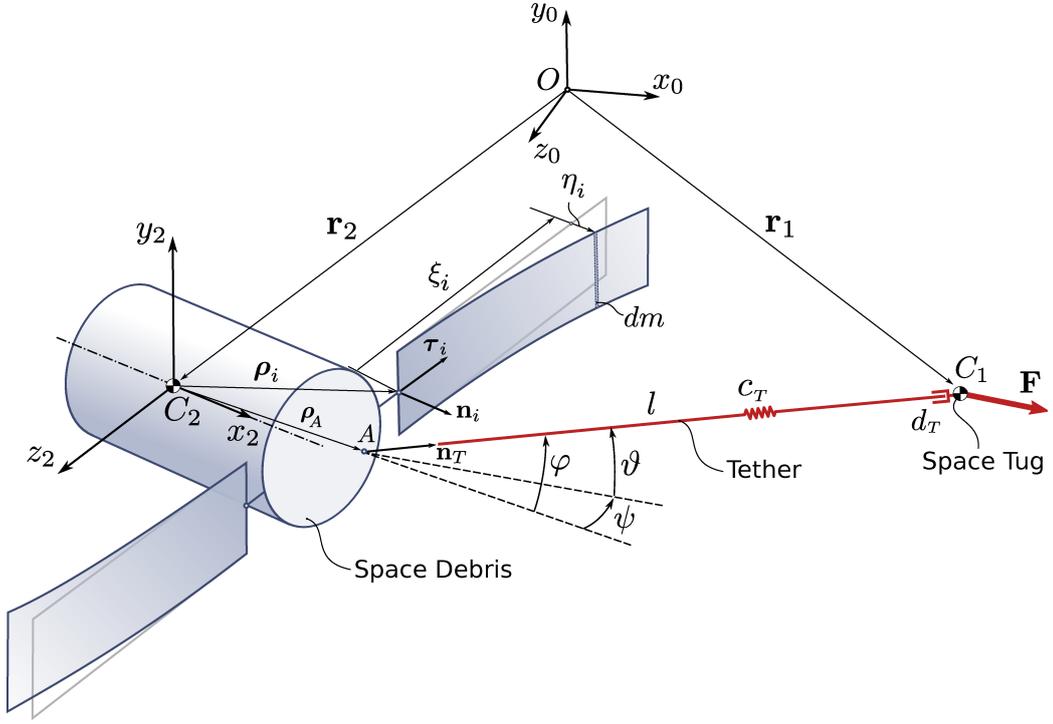


Figure 2: Space debris with flexible appendages

orbital frame $Ox_0y_0z_0$

$$\mathbf{M}_\alpha = \begin{bmatrix} c_{\alpha_2}c_{\alpha_3} & c_{\alpha_1}s_{\alpha_3} + c_{\alpha_3}s_{\alpha_2}s_{\alpha_1} & s_{\alpha_3}s_{\alpha_1} - c_{\alpha_3}c_{\alpha_1}s_{\alpha_2} \\ -c_{\alpha_2}s_{\alpha_3} & c_{\alpha_3}c_{\alpha_1} - s_{\alpha_2}s_{\alpha_3}s_{\alpha_1} & c_{\alpha_1}s_{\alpha_2}s_{\alpha_3} + c_{\alpha_3}s_{\alpha_1} \\ s_{\alpha_2} & -c_{\alpha_2}s_{\alpha_1} & c_{\alpha_2}c_{\alpha_1} \end{bmatrix}$$

where $c_{\alpha_i} = \cos \alpha_i$, $s_{\alpha_i} = \sin \alpha_i$ ($i = 1, 2, 3$); \mathbf{l} is a vector of the AC_1 line in $C_2x_2y_2z_2$

$$\mathbf{l} = \begin{bmatrix} \cos \vartheta \cos \psi \\ \cos \vartheta \sin \psi \\ -\sin \vartheta \end{bmatrix} l$$

The angular velocity of the space debris is expressed as [25]

$$\begin{cases} \omega_{2x} &= \dot{\alpha}_1 \cos \alpha_2 \cos \alpha_3 + \dot{\alpha}_1 \sin \alpha_3 \\ \omega_{2y} &= -\dot{\alpha}_1 \cos \alpha_2 \sin \alpha_3 + \dot{\alpha}_2 \cos \alpha_3 \\ \omega_{2z} &= \dot{\alpha}_1 \sin \alpha_2 + \dot{\alpha}_3 \end{cases}$$

2.3. Kinetic energy of the system

The kinetic energy of the rigid bodies (debris and tug) is

$$2K_b = m_1 |\mathbf{v}_1|^2 + m_1 |\mathbf{v}_2|^2 + \boldsymbol{\omega}_2^T \mathbf{J}_2 \boldsymbol{\omega}_2, \quad (2)$$

To describe the motion of flexible appendages the normal-mode expansion technique is used. The deformation of the flexible appendage i as a function of ξ_i (fig. 2) is defined as

$$\eta_i = \sum_{j=1}^{\infty} f_j(\xi_i) q_{ij}(t) \quad (3)$$

where $f_j(\xi_i)$ is a j mode shape function for j natural frequency, $q_{ij}(t)$ is a generalized coordinate corresponded to j mode. Mode shape functions for the fixed-free beam have the following form [26]

$$f_j(\xi_i) = C_j \left[\cosh \frac{k_j \xi_i}{l_i} - \cos \frac{k_j \xi_i}{l_i} - a_j \left(\sinh \frac{k_j \xi_i}{l_i} - \sin \frac{k_j \xi_i}{l_i} \right) \right]$$

where

$$a_j = \frac{\cos k_j + \cosh k_j}{\sin k_j + \sinh k_j}$$

and

$$k_j^2 = \omega_j \sqrt{\frac{\mu_i l_i^4}{E_i J_i}}$$

where k_j is square of the nondimensional natural frequency, ω_j is a dimensional natural frequency. C_j is an unessential constant multiplier that is taken so that $f_j(l_i) = 1$, where l_i is a length of the i flexible appendage.

For the fixed-free beam k_j defined by the equation [26]

$$\cosh k_i \cos k_i = -1$$

First three nondimensional frequencies are

$$k_1^2 = 3.51, \quad k_2^2 = 22.03, \quad k_3^2 = 61.70$$

Now we can write expression for the kinetic energy of the flexible appendage. The velocity of the mass element dm of the flexible appendage i in frame $Ox_0y_0z_0$ is

$$\mathbf{v}_{\eta i} = \mathbf{v}_2 + \boldsymbol{\omega}_2 \times \left(\boldsymbol{\rho}_i + \boldsymbol{\tau}_i \xi_i + \mathbf{n}_i \sum_{j=1}^{\infty} f_{ij}(\xi_i) q_{ij}(t) \right) + \mathbf{n}_i \sum_{j=1}^{\infty} f_{ij}(\xi_i) \dot{q}_{ij}(t) \quad (4)$$

The kinetic energy of the flexible appendage is

$$2K_{fi} = \int_0^{l_i} |\mathbf{v}_{\xi i}|^2 dm$$

and the total kinetic energy of the system is

$$2K = 2K_b + \sum_{i=1}^{n_f} 2K_{fi} \quad (5)$$

where n_f is a number of the flexible appendages.

2.4. Generalized forces

The tether tension force acting on the space debris is expressed as

$$\mathbf{T} = \begin{cases} [c_t(l - l_0) + d_t \dot{l}] \mathbf{n}_T, & l > l_0 \\ 0, & l \leq l_0 \end{cases} \quad (6)$$

where c_t is a tether stiffness, l_0 is a tether free length, d_t is a tether damping.

The thruster force vector of the space tug in the frame $Ox_0y_0z_0$ is $\mathbf{F} = (F_x, F_y, F_z)^T$.

The right hand sides of the equations (1) are written as

$$Q_i = \frac{\partial \mathbf{r}_A}{\partial s_i} \cdot \mathbf{T} - \frac{\partial \mathbf{r}_1}{\partial s_i} \cdot \mathbf{T} + \frac{\partial \mathbf{r}_1}{\partial s_i} \cdot \mathbf{F} - \sum_{j=1}^{n_f} \frac{\partial \Pi_{fj}}{\partial s_i} \quad (7)$$

where $\mathbf{r}_A = \mathbf{r}_2 + \mathbf{M}_\alpha \boldsymbol{\rho}_A$. The potential energy of flexible appendage j is [26]

$$\Pi_{fj} = \frac{1}{2} \int_0^{l_j} E_j J_j \frac{\partial^2 \eta_j}{\partial \xi_j^2} d\xi_j \quad (8)$$

In this paper we suppose that the thrust force acting on the tug provides torque much greater magnitude compared to the gravitational torque. Therefore, we do not include in the gravitational potential energy terms from the interaction between the Earth and the tethered system.

For $\boldsymbol{\rho}_A = (x_A \ y_A \ z_A)^T$ and $\mathbf{F} = (F, 0, 0)^T$ the generalized forces for the generalized coordinates $x_2, y_2, z_2, \alpha_1, \alpha_2, \alpha_3, \psi, \vartheta, l$ are

$$\begin{aligned} Q_{x_2} &= F, Q_{y_2} = 0, Q_{z_2} = 0, Q_{\alpha_1} = 0 \\ Q_{\alpha_2} &= F(\sin \alpha_2 (y_A \sin \alpha_3 - x_A \cos \alpha_3) + z_A \cos \alpha_2 - \\ &\quad l(\sin \alpha_2 \cos \theta \cos(\alpha_3 + \psi)) + \cos \alpha_2 \sin \theta) \\ Q_{\alpha_3} &= -F \cos \alpha_2 (l \cos \theta \sin(\alpha_3 + \psi) + x_A \sin \alpha_3 + y_A \cos \alpha_3) \\ Q_{\psi} &= Fl \cos \alpha_2 \cos \vartheta \sin(\alpha_3 + \psi) \\ Q_{\vartheta} &= Fl(\cos \alpha_2 \sin \vartheta \cos(\alpha_3 + \psi) + \sin \alpha_2 \cos \vartheta), \\ Q_l &= F(\cos \alpha_2 \cos \vartheta \cos(\alpha_3 + \psi) - \sin \alpha_2 \sin \vartheta). \end{aligned}$$

3. Analysis and numerical examples

3.1. Analysis and numerical examples

The aim of this work is to study the motion of a space debris during the initial phase of the orbital transportation. We show the interference between

the tether vibrations and the vibrations of flexible appendages that can lead to the mission failure. We suppose that the passive spacecraft (space debris) has two flexible appendages and the spacecraft already connected to the space tug. Let us consider several numerical examples of the relative motion of the passive spacecraft and the space tug.

The amplitude of the oscillation decreases with the increase in the oscillation frequency then for the sake of simplicity only one shape function for each flexible appendage (panel) is taken, i.e. deformation of the panel has the form (3)

$$\xi_i = f_1(\eta_i)q_{i1}(t)$$

There are three cases considered. In the Case 1 the natural frequency of the tether differs from the natural frequency of the solar panels of the passive spacecraft. In the Case 2 the natural frequency of the tether close to the natural frequency of the solar panels of the passive spacecraft and the tether attachment point is located close to the flexible appendages. In this case we simulate a situation of structural failure of the solar panel. The Case 3 differs from the Case 2 in that the attachment point of the tether is located far from flexible appendages than in the Case 2.

3.2. Parameters of the system and initial conditions

The parameters of the passive spacecraft and the space tug are presented in table 1. The tether is assumed to be made of Kevlar like material. Sectional area of the tether is 7.8 mm^2 , length is 50 m and tether damping is $d_t = 0$.

At $t = 0$ passive spacecraft rotates around its x-axis with $1^\circ/s$ and the tether starts pull the spacecraft at a sharp angle φ relative to the x-axis of

Table 1: Parameters of the space tug and the passive spacecraft

Parameter	Value	Parameter	Value
Space tug mass, kg	500	Debris mass, kg	3000
			$J_{2x} = 2000$
Tug thruster force, N	20	Debris moments of inertia, $kg \cdot m^2$	$J_{2y} = 8000$ $J_{2z} = 8000$
Tether stiffness (Kevlar with sectional area 7.8 mm^2), N/m	15586	Tether length, m	50
Solar panel length, m	5	Bending stiffness of the solar panel, EJ , $N \cdot m^2$	1840
Tether attachment point, \mathbf{r}_T	$[1, 0, 0.3]$	Mass per unit length of the solar panel – μ , kg/m	10

the spacecraft. All cases start with the following initial conditions

$$\begin{aligned}
 x_0 = y_0 = z_0 = 0, \quad \dot{x}_0 = \dot{y}_0 = \dot{z}_0 = 0, \\
 \alpha_1 = \alpha_2 = \alpha_3 = \psi_0 = \vartheta_0 = 0, \\
 \dot{\alpha}_1 = 0.05, \dot{\alpha}_2 = \dot{\alpha}_3 = \dot{\psi}_0 = \dot{\vartheta}_0 = 0, \dot{l}_0 = 0.
 \end{aligned}$$

3.3. Case 1: motion of the debris with fore-mounted flexible appendages

To give some justification for using only one tone to describe the deformation of flexible appendages we consider the vibration of flexible appendages taking two tones from the expression (3). Fig. 3 shows a plot of the variables q_{11} , q_{12} as a function of time. From this figure it can be seen that amplitude of the first mode (dashed line) an order of magnitude more than the second mode amplitude. With this result we consider first tone of the oscillations.

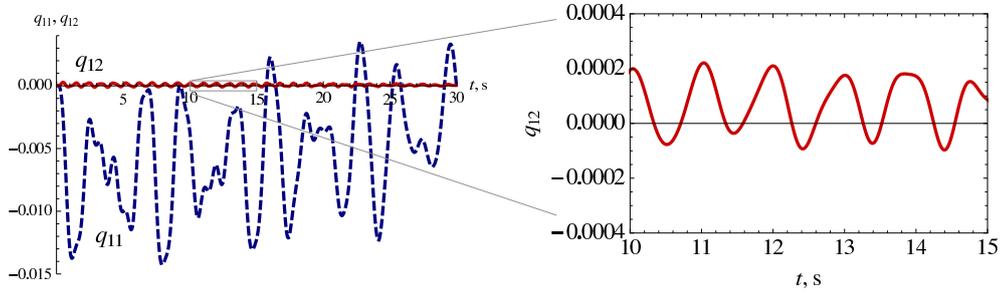


Figure 3: Two tones of the oscillations of flexible appendage

At first we consider a case when the natural frequency of the tether differs from the natural frequency of the solar panels of the space debris. The natural frequency of the tether is higher than the frequency of the flexible appendages ($EJ = 1840 \text{ N} \cdot \text{m}^2$, $c_t = 15586 \text{ N/m}$).

Fig. 4 shows the vibrations of the flexible appendages q_1, q_2 of the passive spacecraft, the tension force of the tether T and the angle φ .

Note that in this case the vibrations of the flexible appendages haven't a significant influence on the tether vibrations and on the attitude motion of the debris.

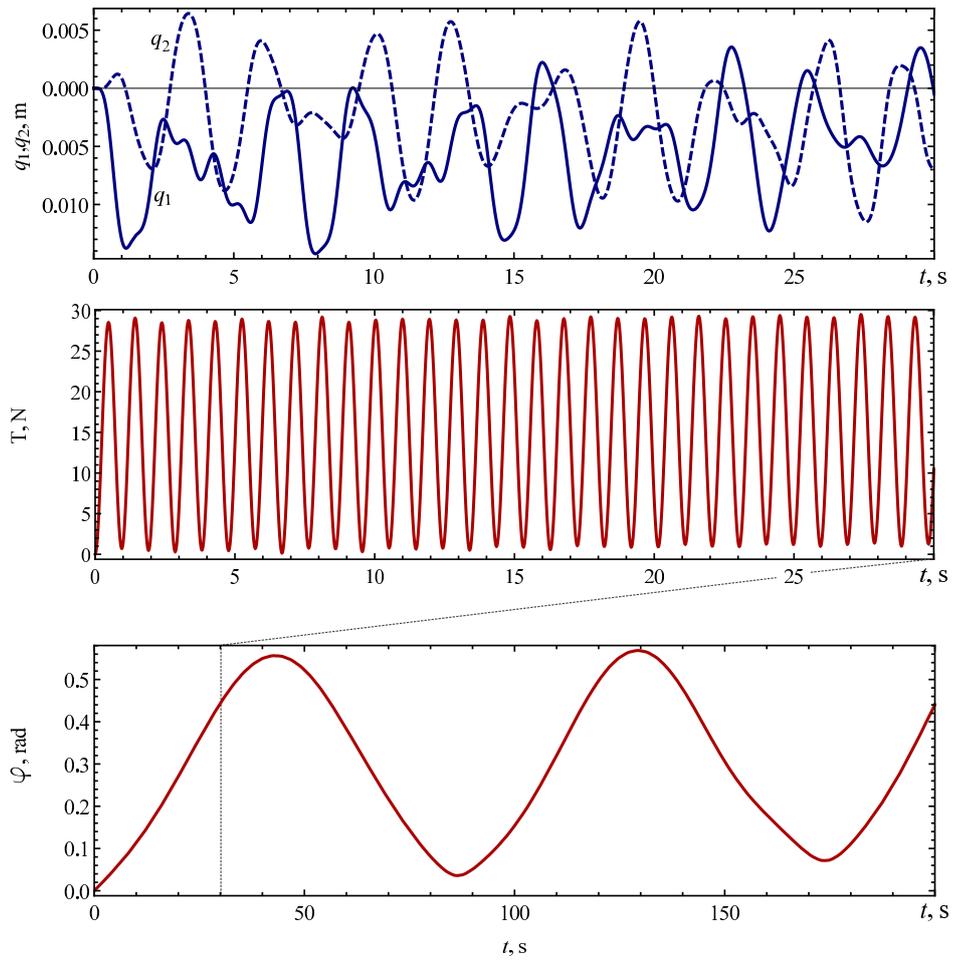


Figure 4: The vibrations of the solar panels, the tension force of the tether and the angle φ for the Case 1

3.4. Case 2: motion of the space debris with fore-mounted flexible appendages

In this case the natural frequency of the tether close to the natural frequency of the solar panels of the passive spacecraft ($EJ = 1840 \text{ N} \cdot \text{m}^2$, $c_t = 1558 \text{ N/m}$).

Fig. 5 shows the vibrations of the flexible appendages of the passive spacecraft the tension force of the tether and the angle φ for the Case 2.

In this case we assume that the each solar panel has a breaking strain $|q^b|$, denoted in the fig. 5 by the red dashed lines.

At $t = t_1 \approx 15\text{s}$ the deformation of the panel 2 q_2 reach the breaking strain causing structure failure. We suppose that at t_1 space debris loose it's solar panel and continues motion with only one panel. The motion of the unbalanced debris can lead to the breakdown of the next solar panel. At $t_2 \approx 60\text{s}$ panel 1 breaks of too.

Fig 5 also demonstrates the mutual influence of the panel's oscillation and the oscillation of the tether. Unlike the Case 1, the amplitude of the tether vibrations is influenced by the vibrations of the solar panels and vice versa.

3.5. Case 3: motion of the space debris with aft-mounted flexible appendages

In this case just as in the case 2 the natural frequency of the tether close to the natural frequency of the solar panels of the debris, but the tether attachment point is located farther from the flexible appendages than in the Case 2. In Fig. 6 we see that the structural failure occurred earlier than in the Case 2.

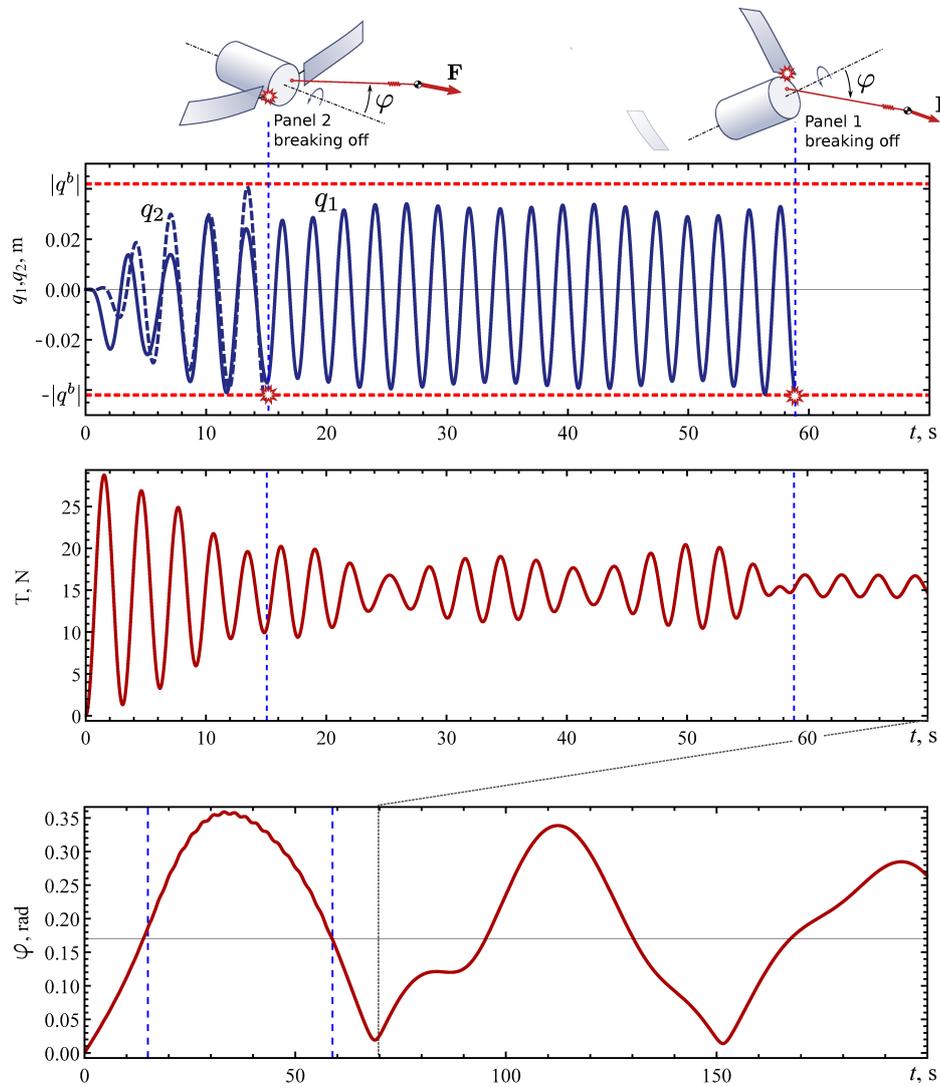


Figure 5: The vibrations of the solar panels, the tension force of the tether and the angle φ for the Case 2

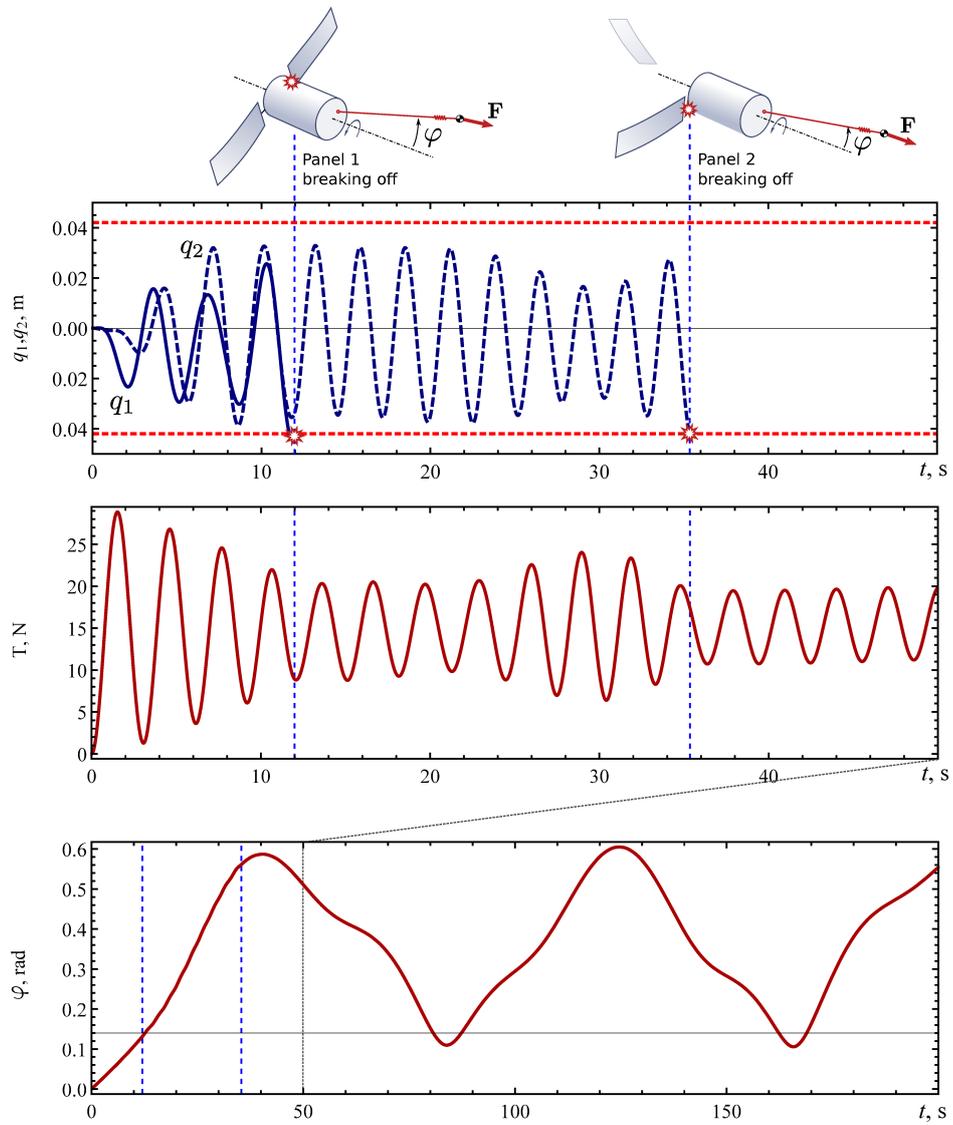


Figure 6: The vibrations of the solar panels, the tension force of the tether and the angle φ for the Case 3

4. Conclusion

The mathematical model of the system consisting of the space tug the tether and the large space debris with flexible appendages is developed. Several numerical examples show that the space debris with flexible appendages can affect to the safety of the transportation process. To reduce risk of the structure failure the large amplitude vibrations of the flexible appendages should be avoided. The properties of the tether should be chosen taking into account the properties of the flexible appendages – the natural frequency of the tether shouldn't induce large vibrations of the flexible appendages. Proposed mathematical model can be used to analyze active debris removal of the large passive spacecraft with flexible appendages.

Future research should be directed toward investigating the influence of the variations in the parameter of the system on the safety of the transportation process including the analysis of the influence of uncertainties in the parameters of the space debris with flexible appendages. It allows to obtain the parameter space of the tug-tether system for the safe active debris removal mission.

Note 2

Acknowledgment

This work was supported by Ministry of education and science of Russia (Contract No. 9.540.2014/K).

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