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Features of Rotational Motion of a Spacecraft Descending in the Martian Atmosphere

V. S. Aslanov and A. S. Ledkov
Korolev Samara State Aerospace University, Samara, Russia

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Abstract—Angular motion at atmospheric entry is studied in the paper for a spacecraft with a bi-harmonic moment characteristic. Special attention is given to the case when the spacecraft possesses two stable balanced positions, and, hence, it can oscillate in dense atmospheric layers in the ranges of small or large angles of attack. The averaged equations of spacecraft motion are derived, which allow one to increase the speed of calculations by several orders of magnitude. A real example is presented, which concerns a spacecraft specially designed for descending in the Martian atmosphere.

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1. FORMULATION OF THE PROBLEM

It is agreed that one of main causes, resulting in anomalous behavior of spacecraft at atmospheric entry, is the parametric resonance [1, 2]. It arises in the presence of a small mass-inertial and aerodynamic asymmetry, when the motion relative to the center of mass depends on two angular variables: the spatial angle of attack and the angle of spinning. When the frequency of oscillation of the angle of attack and the average angular velocity of spinning become multiple to the ratio of prime integers under an effect of disturbances, the resonance arises. The resonance, as a phenomenon of considerable change in the amplitude of oscillations, can also arise in the absence of asymmetry, when the motion depends on a single angular variable, the spatial angle of attack, while the aerodynamic restoring moment coefficient $m_\alpha(\alpha)$ vanishes at three points on the segment $[0, \pi]$. In this case, on the phase portrait $\dot{\alpha} = \dot{\alpha}(\alpha)$ one can observe three regions separated by a separatrix [3]. Under an effect of disturbances, such as dynamic pressure variation at spacecraft descent in the atmosphere, the phase trajectory can intersect the separatrix, thus transferring from one region to another, which is accompanied by a jump change of the oscillation amplitude and represents a resonance [4]. For descent in the rarefied Martian atmosphere the blunt-shaped bodies of small elongation are used, which provides for effective drag. Such bodies, depending on their mass configuration, can possess, along with two balancing positions of the angle of attack: $\alpha^* = 0, \pi$, also the third equilibrium position: $\alpha^* \in (0, \pi)$. Figure 1 presents the segmental-conic body and dependencies of a restoring aerodynamic moment coefficient $m_\alpha(\alpha)$ on the spatial angle of attack for various positions of the center of mass measured from the body's nose ($\bar{x}_T = x_T/l$, where l is the characteristic size of a body), found using the Newton's shock theory.

To approximate the restoring moment coefficient we make use of a bi-harmonic dependence of the form

$$m_\alpha(\alpha) = a\sin\alpha + b\sin 2\alpha. \qquad (1.1)$$

For the considered class of spacecraft the position $\alpha = 0$ is stable; therefore, the derivative of the restoring moment coefficient with respect to the angle of attack at this point is negative

$$(a\cos\alpha + 2b\cos 2\alpha)|_{a=0} < 0,$$

or

$$2b < -a. \qquad (1.2)$$

And if there exists an intermediate balancing position on the interval $(0, \pi)$, then

$$m_\alpha(\alpha) = a\sin\alpha + b\sin 2\alpha$$
$$= \sin\alpha(a + 2b\cos\alpha) = 0,$$

which holds true, if

$$|2b| > |a|. \qquad (1.3)$$

It is obvious that inequalities (1.2) and (1.3) are valid simultaneously at $b < 0$. Note that the dependencies $m_\alpha(\alpha)$ presented in Fig. 1 satisfy conditions (1.2) and (1.3).

The problem is stated to demonstrate the possibility of appearance of resonances for axi-symmetric bodies intended for entering the Martian atmosphere, to find the motion stability conditions, to obtain the averaged equations of disturbed motion, and to construct the procedure for calculating the upper and lower estimates of motion parameters with the use of the averaged equations.

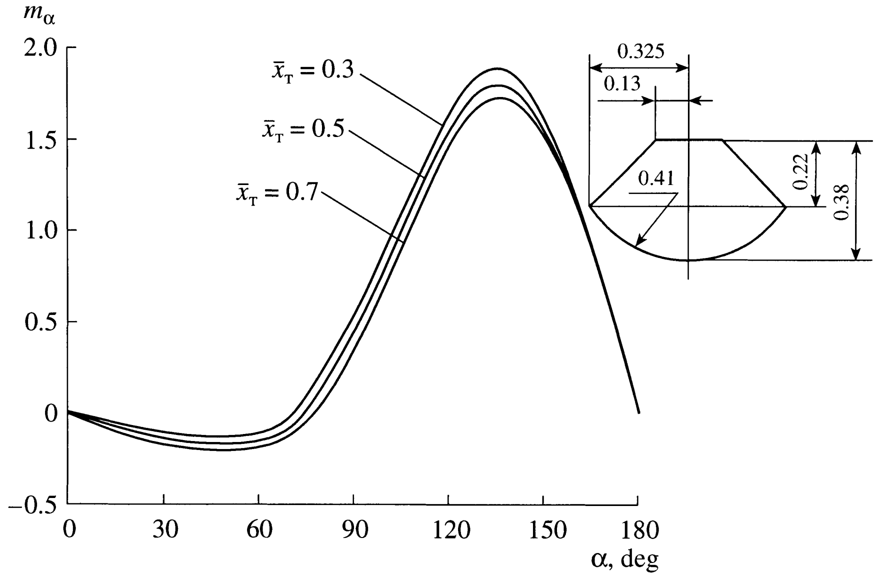

Fig. 1. The coefficient of the aerodynamic restoring moment versecs the spatial augle of attack at varicus positions of the center of mass.

2. EQUATIONS OF MOTION AND THE SYSTEM'S PHASE PORTRAIT

We write the equations of three-dimensional motion of an axi-symmetric body at descent in the atmosphere in the following form [2]:

$$\begin{gathered}\ddot{\alpha} + \frac{(G - R\cos\alpha)(R - G\cos\alpha)}{\sin^3\alpha} \\ -M_\alpha(\alpha, z) = -m_z(z)\dot{\alpha}, \\ \dot{R} = -\varepsilon m_x(z)R = \varepsilon\Phi_R(z), \\ \dot{G} = -\varepsilon\{m_y(z)G + [m_x(z) - m_y(z)]R\cos\alpha\} \\ = \varepsilon\Phi_G(\alpha, z), \\ \dot{V} = -c_{x\alpha}(\alpha)\frac{qS}{m} - g\sin\theta = \varepsilon\Phi_V(\alpha, z), \\ \dot{\theta} = -\frac{\cos\theta}{V}\left(g - \frac{V^2}{R_P + H}\right) = \varepsilon\Phi_\theta(\alpha, z), \\ \dot{H} = V\sin\theta = \varepsilon\Phi_H(\alpha, z),\end{gathered} \tag{2.1}$$

where $z = (R, G, V, \theta, H)$ is the vector of slowly varying parameters; α is the spatial angle of attack, ε is a small parameter, R and G are, to an accuracy of a multiplier, the projections of the angular momentum vector onto the longitudinal axis and onto the velocity direction, respectively; V is the spacecraft motion velocity, θ is the trajectory inclination angle, H is the flight altitude, g is the acceleration of gravity, $c_{x\alpha}(\alpha)$ is the drag force coefficient, $q = \rho V^2/2$ is the dynamic pressure, ρ is the density of the atmosphere, S is the middle cross section area, m is the spacecraft mass, $M_\alpha = m_\alpha qSL/I$ is the restoring moment to an accuracy of a multiplier (I is the transverse moment of inertia of the body, and L is its characteristic size), R_P is the planet's radius; $\varepsilon m_x(z)$, $\varepsilon m_y(z)$, and $\varepsilon m_z(z)$ are the projections of a small damping moment onto the axes of the right-handed coordinate system $Oxyz$ chosen in such a manner that the Ox axis is directed along the spacecraft's axis of symmetry, the Oy axis lies in the plane formed by Ox and velocity vector V.

It should be noted that the right-hand sides of the equations of system (2.1) can also be written in a more complicated form, such as that in [1, 2]. However, of principal significance is here the circumstance that the right-hand sides are functions of only one "fast" variable, the spatial angle of attack α. We present the system (2.1) in a short-cut form:

$$\ddot{\alpha} + F(\alpha) = -\varepsilon m_z(z)\dot{\alpha}, \quad \dot{z} = \varepsilon\Phi_z(\alpha, z). \tag{2.2}$$

Disturbed system (2.2) for $\varepsilon = 0$ is reduced to the undisturbed system with a single degree of freedom. The evolution of motion parameters proceeds under an effect of disturbances arising due to small damping moments and dynamic pressure variability. Now we should find the relationship between the presence of three balancing positions of a bi-harmonic characteristic (1.1), under conditions (1.2) and (1.3), and the existence of stable and unstable equilibrium positions on the phase portrait of an undisturbed system obtained from (2.2) for $\varepsilon = 0$:

$$\ddot{\alpha} + F(\alpha) = 0, \tag{2.3}$$

where

$$F(\alpha) = \frac{(G - R\cos\alpha)(R - G\cos\alpha)}{\sin^3\alpha} - A\sin\alpha - B\sin 2\alpha, \quad (2.4)$$

$$A = \frac{aSL}{I}q, \quad B = \frac{bSL}{I}q.$$

Equation (2.3) has the integral of energy

$$\dot{\alpha}^2/2 + W(\alpha) = E, \quad (2.5)$$

$$W(\alpha) = \int F(\alpha)d\alpha = W_g(\alpha) + W_r(\alpha), \quad (2.6)$$

where $W_g(\alpha) = (G^2 + R^2 - 2GR\cos\alpha)/[2\sin^2\alpha]$, $W_r(\alpha) = A\cos\alpha + B\cos^2\alpha$.

There is one-to-one correspondence between the values of variable $u = \cos\alpha$ on the segment [–1, +1] and the values of angle α on the segment $[0, \pi]$. With regard to replacement $u = \cos\alpha$ integral of energy (2.5) can be written as

$$\dot{u}^2/[2(1-u^2)] + W_g(u) + W_r(u) = E, \quad (2.7)$$

where $W_g(u) = (G^2 + R^2 - 2GRu)/[2(1 - u^2)]$, $W_r(u) = Au + Bu^2$.

Now let us present (2.7) as follows: $\dot{u}^2 - f(u) = 0$, where

$$f(u) = 2(1-u^2)(E - Au - Bu^2) + 2GRu - G^2 - R^2. \quad (2.8)$$

The character of a phase portrait of the system described by equation (2.7) is determined by the form of potential function $W(u)$. In particular, the number and position of extreme points of this function determine the number and type of singular points. The stable point of the center type corresponds to the minimum, and the unstable point of the saddle type to the maximum. The behavior of function $W(u) = W_g(u) + W_r(u)$ for various combinations of R, G, A, and B parameters was studied in [2]; we present here only the results of this study. The potential function $W(u)$ has no inflection points on the (–1, 1) interval, provided that

$$B \geq -[\min_{-1 \leq u \leq 1}(0.5W_g''(u))] \equiv B^*, \quad (2.9)$$

since its second derivative $W''(u) = W_g''(u) + W_r''(u)$ with respect to variable u is non-negative. This implies that there is no saddle singular point on the phase portrait. According to (2.9), quantity B^* is always negative. For $R = G = 0$ function $W_g''(u)$ degenerates; therefore, $B^* = 0$, and condition (2.9) assumes the form of $B \geq 0$. The saddle point will also be absent at a relatively small value of coefficient b as compared to a (the motion close to the Lagrangian case). Really, if

$$|b| \leq 0.5|a|, \quad (2.10)$$

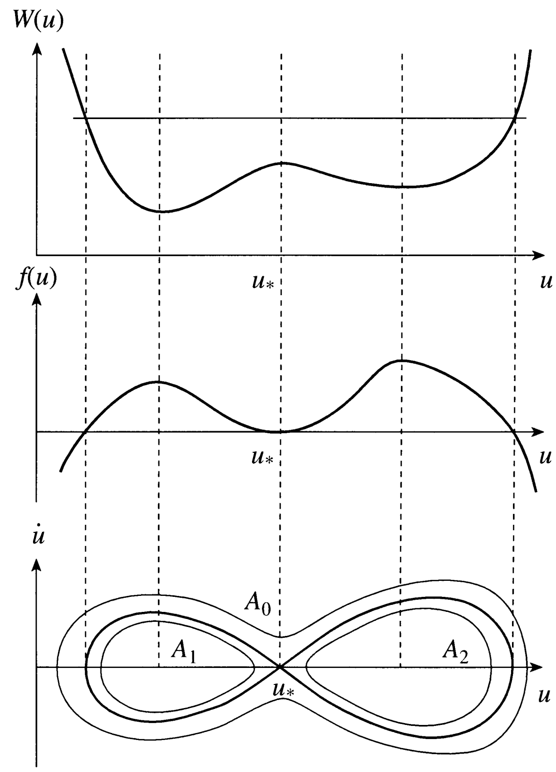

Fig. 2. Phase portrait.

then function $W_r''(u)$ has one and the same sign throughout the interval, and, hence, the derivative $W'(u) = W_g'(u) + W_r'(u)$ vanishes at a single point, and function $W(u)$ has a single extremum (minimum). Condition (2.10) contradicts condition (1.3), and, if condition (2.9) is not met, function $W(u)$ can have two minima and one maximum on the (–1, 1) interval, which corresponds to the presence of an unstable singular point of the saddle type on the phase portrait. It is obvious that the aforementioned situation arises upon satisfying the condition

$$W'(u_{*1})W'(u_{*2}) < 0, \quad (2.11)$$

where u_{*1} and u_{*2} are the roots of the equation $W''(u) = 0$.

When condition (2.11) is met, the phase plane is divided by a separatrix into three regions: the outer region A_0 and two inner regions A_1 and A_2. If $E > W_*$, where W_* is the value of $W(u)$ at a saddle point $u = u_*$, then the motion proceeds in the outer region A_0 (Fig. 2). In the opposite case ($E < W_*$) the motion can occur in any of innner regions A_1 or A_2, depending on the initial conditions. The equality $E = W_*$ corresponds to the motion along the separatrix.

3. STABILITY OF PERTURBED MOTION

Under the action of perturbations, arising during spacecraft descent in the atmosphere, the phase trajectory, while remaining in one of the regions, either moves apart from a separatrix or approaches it. In the first case the trajectory is "immersed" deeper into a given region, and in the second case it is "pushed out" from it. Accordingly, we will refer to regions A_0, A_1, and A_2 as stable or unstable. The motion can start either in outer region A_0 or in any of inner regions A_1 and A_2. If the region in which the motion has begun is unstable, the phase trajectory intersects the separatrix in some finite time. Obviously, two situations can take place at the separatrix intersection instant: 1) two regions are unstable and one is stable, and 2) on the contrary, one region is unstable and two are stable. In the first case the motion continues in the stable region only, and in the second case the further behavior of a trajectory depends on the current phase of the angle of attack. If the phase is not determined, to fall into any region is of a random character. The author of [4] proposes to use, for choosing the continuation of motion, the concept of probability of "capture" into each region. This probability is determined on the basis of calculating the areas of regions encompassed by a separatrix. Analytical finding of these areas is reduced to calculation of improper integrals.

In order to estimate the stability of the regions it is not necessarily to calculate their areas. Under the action of small perturbations the average value of the total energy $\bar{E}$ slowly changes, as well as the value of potential energy W_*, calculated at the saddle point $u = u_*$. For determining the stability it is sufficient to make use of time derivatives of mentioned functions [2]. The inner region (A_1 or A_2) is stable, if the following condition is satisfied near the separatrix:

$$\dot{\bar{E}}(z) < \dot{W}(u_*, z). \tag{3.1}$$

For the outer region A_0 the stability condition is as follows:

$$\dot{\bar{E}}(z) > \dot{W}(u_*, z). \tag{3.2}$$

The value of function (2.8) at the saddle point $u = u_*$ is equal to:

$$f_* \equiv f(u_*, z) = 2(1-u_*^2)[\bar{E}(z) - W(u_*, z)]. \tag{3.3}$$

In the neighborhood of the separatrix $\bar{E}(z) - W(u_*, z) = O(\varepsilon)$, $\dot{u}_*(z) = O(\varepsilon)$, and the differentiation of function (3.3) with respect to time, to an accuracy of quantities of the order of ε^2, gives the following result:

$$\dot{f}_* = 2(1-u_*^2)[\dot{\bar{E}}(z) - \dot{W}(u_*, z)]. \tag{3.4}$$

It follows from (3.4) that the conditions $\dot{f}_* < 0$ and $\dot{f}_* > 0$ correspond, respectively, to conditions (3.1) and (3.2) [3]. Indeed, if in the inner region (A_1 or A_2) the value of polynomial $f(u)$ at point u_* decreases, then this region is stable. In the opposite case the region is unstable, and the phase trajectory will not fall into it at any initial conditions. Similarly, the outer region A_0 will be stable or unstable with increasing or decreasing f_*, respectively.

It follows from energy integral (2.5) that the total energy is equal to potential one $E = W(\alpha_m)$, calculated for the amplitude value of the angle of attack $\alpha = \alpha_m$ (for $\dot{\alpha} = 0$). It is obvious that

$$\bar{E}(z) = W(\alpha_m, z), \tag{3.5}$$

where α_m and z correspond to the averaged equations. We suppose that the averaged equations of motion corresponding to system (2.2) are obtained. Let us calculate the derivatives $\dot{\bar{E}}(z)$ and $\dot{W}(\alpha_*, z)$ in virtue of the averaged equations:

$$\dot{\bar{E}}(z) = \left.\frac{\partial W}{\partial \alpha}\right|_{\alpha=\alpha_m} \dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_m} \dot{z}$$

$$= F(\alpha_m, z)\dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_m} \dot{z}$$

and

$$\dot{W}(\alpha_*, z) = \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_*} \dot{z}.$$

Now we introduce the criterion which determines stability of the perturbed motion in the separatrix neighborhood:

$$\Lambda \equiv F(\alpha_m, z)\dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha_*}^{\alpha_m} \dot{z}, \tag{3.6}$$

and then, finally, we can write the stability conditions: for the inner regions A_1 and A_2 (3.1)

$$\Lambda < 0 \tag{3.7}$$

and for outer region A_0 (3.2)

$$\Lambda > 0. \tag{3.8}$$

4. AVERAGED EQUATIONS AND MODELING OF THE PERTURBED MOTION

The stability criterion Λ is a function of the amplitude value of the angle of attack; therefore, it is expedient to write the averaged system for this angle directly. In addition, the numerical modeling of the perturbed motion is convenient to be performed with the use of the averaged equations. For the known solution of unperturbed system (2.3) [2], we write, by means of V.M. Volosov's method [5] for perturbed system (2.1), the averaged equations, having chosen the maximum angle of attack as an amplitude α_m:

$$\dot{\alpha}_m = \frac{2\varepsilon}{TF(\alpha_m)}\Big[m_z I_1 + (R^2 m_x + G^2 m_y) I_2 - 2GRm_y I_3 - R^2(m_x - m_y) I_4 + (A_q I_5 + B_q I_6)\Phi_q - \frac{G - R\cos(\alpha_m)}{\sin^2(\alpha_m)}(m_x - m_y) R I_5\Big]$$

$$- \frac{\varepsilon}{F(\alpha_m)}\Big[\frac{R - G\cos(\alpha_m)}{\sin^2(\alpha_m)} m_x R + \frac{G - R\cos(\alpha_m)}{\sin^2(\alpha_m)} m_y G + (A_q\cos(\alpha_m) + B_q\cos^2(\alpha_m))\Phi_q\Big],$$

$$\dot{R} = \varepsilon m_x R, \quad \dot{G} = \varepsilon\Big[m_x G + \frac{2}{T}(m_x - m_y) R I_5\Big],$$

$$\dot{V} = -\left[\frac{2}{T}\int_{\alpha_{\min}}^{\alpha_m} \frac{c_{x\alpha}(\alpha) d\alpha}{\sqrt{2(E - W(\alpha))}}\right]\frac{qS}{m} - g\sin\theta = \varepsilon\Phi_V(\alpha_m, z), \tag{4.1}$$

$$\dot{\theta} = -\frac{\cos\theta}{V}\left(g - \frac{V^2}{R_P + H}\right) = \varepsilon\Phi_\theta(z),$$

$$\dot{H} = V\sin\theta = \varepsilon\Phi_H(z),$$

$$\varepsilon\Phi_q(\alpha_m, z) = \dot{q} = \frac{d}{dt}(\rho V^2/2) = \varepsilon[\rho V\Phi_V(\alpha_m, z) + \rho_H\Phi_H(z)V^2/2],$$

$$A_q = \frac{dA}{dq}, \quad B_q = \frac{dB}{dq}.$$

Here

$$I_1 = \int_{\alpha_{\min}}^{\alpha_m} \dot{\alpha} d\alpha, \quad I_2 = \int_{\alpha_{\min}}^{\alpha_m} \frac{d\alpha}{\dot{\alpha}\sin^2\alpha},$$

$$I_3 = \int_{\alpha_{\min}}^{\alpha_m} \frac{\cos\alpha d\alpha}{\dot{\alpha}\sin^2\alpha}, \quad I_4 = \int_{\alpha_{\min}}^{\alpha_m} \frac{\cos^2\alpha d\alpha}{\dot{\alpha}\sin^2\alpha}, \tag{4.2}$$

$$I_5 = \int_{\alpha_{\min}}^{\alpha_m} \frac{\cos\alpha d\alpha}{\dot{\alpha}}, \quad I_6 = \int_{\alpha_{\min}}^{\alpha_m} \frac{\cos^2\alpha d\alpha}{\dot{\alpha}}.$$

The integrals I_i can be reduced to complete normal elliptic Legendre integrals of the first, second, and third kind [6]. For this purpose, depending on the form and position of roots of the polynomial $f(u)$ (see Table), one should make use of one of changes of variables [2]. If there exist four real roots, it is necessary to use the change

$$\cos\alpha = u = \frac{u_1(u_2 - u_3) + u_3(u_1 - u_2)\cos^2\gamma}{(u_2 - u_3) + (u_1 - u_2)\cos^2\gamma}, \tag{4.3}$$

and if there are two real and two complex-conjugate roots, then

$$\cos\alpha = u = \frac{(u_2 + u_1\xi) - (u_2 - u_1\xi)\cos\gamma}{(1 + \xi) - (1 - \xi)\cos\gamma}. \tag{4.4}$$

The following designations are introduced in formulas (4.2) and (4.3): $u_1 = \cos\alpha_{\max}$, $u_2 = \cos\alpha_{\min}$, u_3, u_4, and $u_{34} \pm iv$ are the roots of the polynomial $f(u)$; $\xi = \cos\chi_1/\cos\chi_2$, $\tan\chi_1 = (u_1 - u_{34})/v$, $\tan\chi_2 = (u_2 - u_{34})/v$.

When calculating integrals (4.2) it is convenient to make use of the following expression

$$\frac{d\alpha}{\dot{\alpha}} = -\frac{d\gamma}{\beta\sqrt{1 - k^2\sin^2\gamma}},$$

which is obtained from (2.8), (4.3), and (4.4). The values of coefficients k, β, and period T are determined depending on the type of roots:

—four roots are real: change (4.3)

$$k = \sqrt{\frac{(u_1 - u_2)(u_3 - u_4)}{(u_1 - u_3)(u_2 - u_4)}},$$

$$\beta = \sqrt{-\frac{1}{2}B(u_1 - u_3)(u_2 - u_4)}, \quad T = \frac{2K(k)}{\beta};$$

—two roots are real and two ones are complex-conjugate: change (4.4)

$$k = \frac{1}{2}\left(1 - \frac{(u_1 - u_{34})(u_2 - u_{34}) + v^2}{\sqrt{((u_1 - u_{34})^2 + v^2)((u_2 - u_{34})^2 + v^2)}}\right),$$

Positions of the roots of polynomial $f(u)$ (2.8)

Variant	Region	u_1, u_2	u_3, u_4	Type of roots	Formulas
R1	A_2	$u_1 < 1$	$u_4 < u_3 < -1$	u_1, u_2, u_3, u_4 – real roots	(4.3)
R2	A_1	$u_2 > -1$	$u_3 > u_4 > 1$		
R3	A_2	$u_2 < u_1$	$-1 < u_4 < u_3 < u_2 < u_1 < 1$		
R4	A_1		$-1 < u_2 < u_1 < u_4 < u_3 < 1$		
C0	A_0		$u_2 < u_{34} < u_1$	u_1, u_2 – real roots, $u_{3,4} = u_{34} + iv$ – complex-conjugate roots	(4.4)
C1	A_2		$u_{34} < u_2$		
C2	A_1		$u_{34} > u_1$		

$$\beta = \sqrt{-2B}[((u_1 - u_{34})^2 + v^2)((u_2 - u_{34})^2 + v^2)]^{1/4},$$

$$T = \frac{4K(k)}{\beta},$$

where $K(k)$ is the complete elliptic integral of the first kind.

From the point of view of calculation, numerical integration of averaged equations (4.1) for any region differs only in numbering the roots of the polynomial $f(u)$, and in formulas (4.2) for calculating the integrals I_i. In addition, the use of averaged equations (4.1) allows one to increase by an order of magnitude the speed of calculation as compared to complete system (2.1).

On the basis of performed analysis of the disturbed motion of a body with a bi-harmonic moment characteristic, one can offer the following procedure of calculating the upper and lower estimates of motion parameters with using the averaged equations. Note that, if on the phase portrait there exist three regions: A_0, A_1, and A_2, then in the separatrix neighborhood three versions of mutual position of roots of the polynomial $f(u)$ are realized (see Table): C0, R4, and R3, respectively. The numerical integration of averaged equations (4.1) performed from the initial point, belonging to one of the regions, till the separatrix intersection instant, which is determined by arising transition version, either R3–C0 or C0–R4. Then, criterion Λ is calculated by formula (3.6) for each of regions A_0, A_1, and A_2, and their stability is estimated in accordance with conditions (3.7) and (3.8). Some obvious facts should be emphasized here. If the outer region A_0 is stable, then regions A_1 and A_2 are unstable, and vice versa. The transition from region A_1 into region A_2 and from region A_2 into A_1 is possible only through A_0. The region, from which the entering to the separatrix took place, is always unstable; therefore, there can be either one or two stable regions. In the first case, the stable region is chosen for further continuation of integration; in the second case the problem has probabilistic character, and for obtaining the upper and lower estimates of the solution the calculation is performed twice for each stable region. In this case, when using the complete system of equations (2.1), it is necessary to perform stochastic modeling with a great number of calculations of trajectories, which requires considerable expenses of computer time.

As an example, we consider the uncontrolled descent in the Martian atmosphere of a hypothetical spacecraft, whose geometrical dimensions are presented in Fig. 1, the mass equals 69 kg, and coefficients $a = 0.657$, $b = -1.152$. The damping moment will be disregarded: $m_x = m_y = m_z = 0$. The descent is carried out with the following initial conditions: $\alpha_m = 165°$, $R_0 = 0.2$ s^{-1}, $G_0 = 0.7$ s^{-1}, $V_0 = 5000$ m/s, $\theta_0 = -15°$, $H_0 = 1.2 \cdot 10^5$ m. The motion

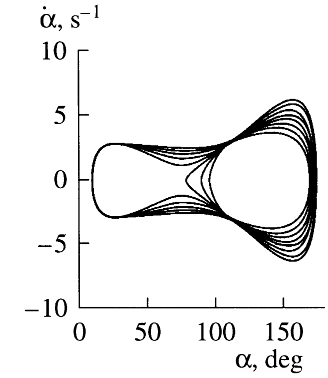

Fig. 3. Transition from region A0 into A1.

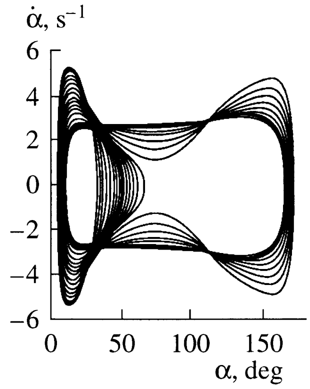

Fig. 4. Transition from region A0 into A2.

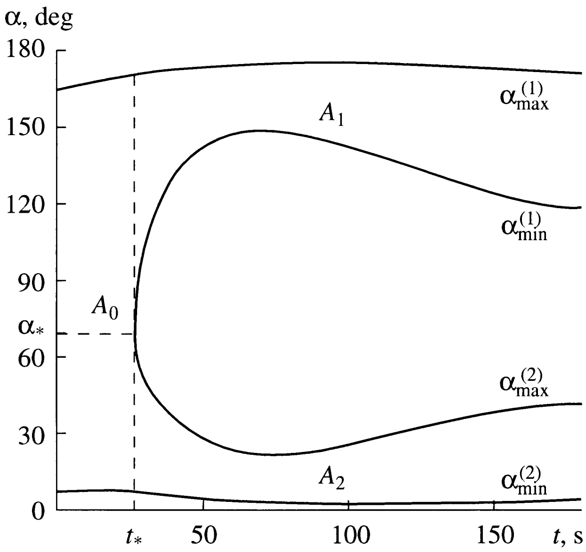

Fig. 5. Amplitudes of jscillations of the angle of attack.

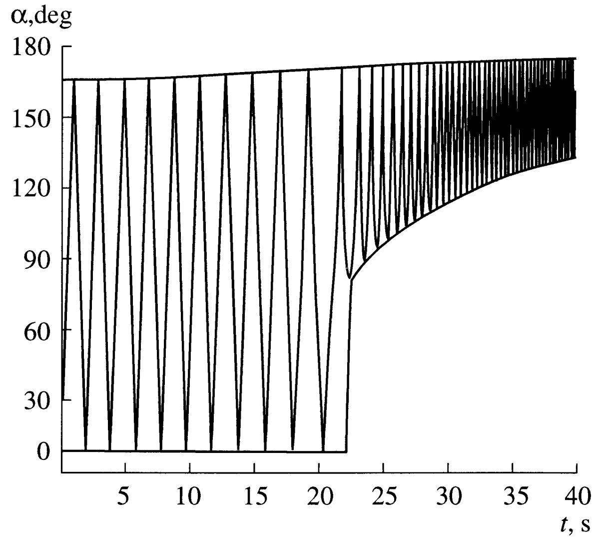

Fig. 6. Comparison of the results of integration for the case of transition from A0 into A1.

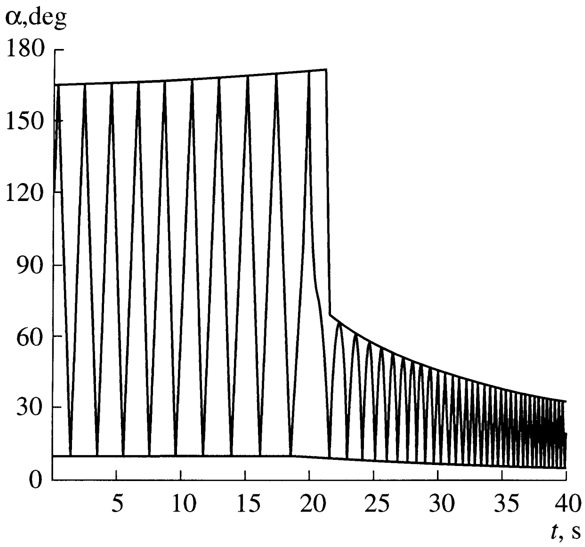

Fig. 7. Comparison of the results of integration for the case of transition from A0 into A2.

begins in the outer region A_0, the transition through the separatrix occurs at $t_* = 22.2$ s, the values of criterion Λ for various regions are equal to: $\Lambda^{(A_0)} = -0.339$, $\Lambda^{(A_1)} = -0.446$, $\Lambda^{(A_2)} = -71.36$. The outer region A_0 is unstable, and regions A_1 and A_2 are stable. Further motion is possible either in region A_1 (Fig. 3), or in A_2 (Fig. 4).

Figure 5 shows the dependences of amplitude values of the angle of attack on the descent time for two possible cases of motion: $A_0 \longrightarrow A_1$ and $A_0 \longrightarrow A_2$.

Figures 6 and 7 give the comparison of the results of numerical integration of initial system (2.1) and of averaged system (4.1).

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