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Gravitational stabilization of a satellite using a movable mass☆

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ABSTRACT

The plane motion of an axisymmetric satellite with a small movable mass on its axis of symmetry is examined, and the equation of the motion of this system in an elliptical orbit is derived. Problems regarding the gravitational stabilization of two diametrically opposite relative equilibrium positions of the satellite in a circular orbit to in-plane perturbations are investigated. A continuous law for controlling the movable mass, which ensures stabilization of the axis of symmetry of the satellite to the local vertical and reorientation of the satellite by moving it from one stable equilibrium position to the other, is constructed using the swing-by technique. A solution is obtained by using the second method of classical stability theory and constructing the corresponding Lyapunov functions. The asymptotic convergence of the solutions with the control obtained is confirmed by the results of numerical simulation of the motion of the system.

The problem of the stability of the relative equilibria and different motions of a satellite about the centre of mass in a Keplerian orbit under the action of gravitational, aerodynamic, and other torques has been the subject of publications by numerous investigators.[1–10] The plane motion of an axisymmetric satellite with a point mass (load) that can move along the axis of symmetry is studied below. The centre of mass of the satellite moves in an orbit under the action of forces of central Newtonian attraction. A law for controlling the satellite is obtained by continuous variation of the distance from the centre of mass of the carrier body to the movable load, according to the principle of swing action.

Swings are usually modelled by a single-mass[11] or two-mass[12,13] pendulum of variable length, and their models can be used to solve applied problems. For example, the swing-by technique has been used[14] to calculate the orbital manoeuvring of a satellite. The problems of gravitational stabilization with respect to in-plane perturbations of the relative equilibrium position of the satellite in a circular orbit and its reorientation are investigated below.

In Section 1 the equation of the plane motion of a satellite with a movable mass about a common centre of mass in a Keplerian orbit of arbitrary eccentricity under the action of a gravitational torque is obtained under the condition that the mass of the load is considerably less than the mass of the carrier. As a consequence the equation of motion of a satellite in a circular orbit is used. In Section 2 a control law is proposed, and the problem of gravitational stabilization (damping of the in-plane oscillations) in the vicinity of a position of relative equilibrium of the satellite, when its axis of dynamic symmetry coincides with the local vertical, is solved. The solution is obtained analytically by constructing the corresponding Lyapunov function. In Section 3 controlling the "swinging" of the satellite in the vicinity of its stable equilibrium position and moving it into a diametrically opposite, asymptotically stable position (turning the satellite through an angle π) are examined. The corresponding Lyapunov function, which reveals the instability of the satellite in the vicinity of the initial equilibrium position and its asymptotic stability in the vicinity of the new equilibrium position, is constructed.

1. The equation of motion of a satellite with a movable mass

Consider the motion of a satellite about its centre of mass in a central Newtonian gravitational field with its centre at the point O. Assuming that the satellite dimensions are small compared with the orbit dimensions, we make the usual assumptions[3] that the motion of the centre of mass of the satellite does not depend on its motion about the centre of mass. The satellite is an axisymmetric rigid body (carrier) of mass m_1, along whose axis of symmetry a point load of mass m_2 can move (Fig. 1). The centre of mass of the carrier lies on its

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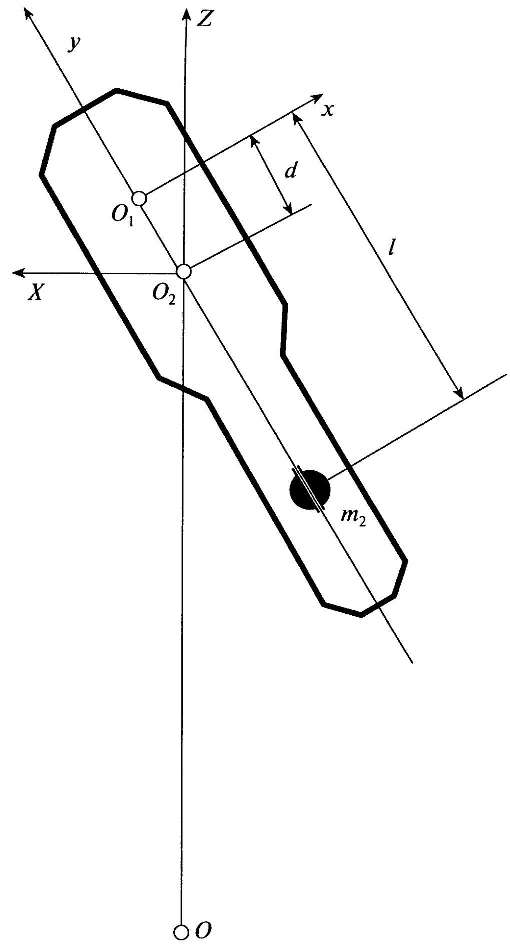

Fig. 1.

dynamic axis of symmetry at the point O_1. We use l and d to denote the distances from the point O_1 to the load and to the centre of mass of the entire satellite O_2, respectively. The following relation holds for them

$$m_1 d = m_2(l - d) \tag{1.1}$$

The orbital system of coordinates O_2XYZ was chosen so that the O_2X axis is directed along a tangent to the orbit, the O_2Y axis is perpendicular to the orbital plane, and the O_2Z axis completes the system of coordinates as a set of three axes at right angles. The axes of the system of coordinates O_1xyz, which is connected to the satellite, coincide with its principal central axes of inertia. The orientation of the connected system of coordinates relative to the orbital system of coordinates is specified using the Euler angles ψ, θ and φ. Suppose A, B and C, where $B < A = C$, are the principal central moments of inertia of the satellite.

We derive the equation of plane motion of a satellite with a movable mass about the common centre of mass under the condition that the mass of the load m_2 is considerably less than the mass of the carrier body m_1:

$$m_2/m_1 = \varepsilon \ll 1 \tag{1.2}$$

Taking into account assumption (1.2), from relation (1.1) we have

$$d = lm_2/(m_1 + m_2) \approx l\varepsilon \tag{1.3}$$

The moments of inertia A_2, B_2 and C_2 of a satellite with a movable load about the axes passing through the common centre of mass O_2 and parallel, respectively, to the axes of the system of coordinates O_1xyz, rigidly connected to the carrier body, are specified, by virtue of relation (1.3), by the equalities

$$A_2 = C_2 = A + m_1 l^2 \varepsilon^2 + m_2 l^2 (1-\varepsilon)^2 \approx A + ml^2, \quad m = m_2(1 - 2\varepsilon)$$
$$B_2 = B = \text{const} \tag{1.4}$$

Only terms which are of the first-order in the small parameter ε have been left in the first relation.

It is well known[3] that the plane motions of a satellite

$$\psi = \pi, \quad \theta = \pi/2, \quad r = \dot{\varphi} + \dot{\nu}, \quad p = q = 0$$

about the centre of mass in an elliptical orbit occur under the action of the gravitational torque

$$M_z = 3n^2 k_1^2 k_2^3 (B_2 - A_2)\sin\varphi\cos\varphi; \quad k_1 = (1-e^2)^{-3/2}, \quad k_2 = 1 + e\cos\nu$$

Here p, q and r are the components of the angular velocity of rotation of the satellite, the dot denotes a derivative with respect to time t, M_z is the gravitational torque about the axis passing through the point O_2 and perpendicular to the orbital plane, n = const > 0 is the mean motion of the centre of mass of the satellite, e is the orbit eccentricity, and ν is the true anomaly. Taking equality (1.4) into account, we write the angular momentum in the form

$$K_z = C_2 r = (A + ml^2)(\dot\varphi + \dot\nu)$$

Then by the theorem of the change in angular momentum,[15] the equation of plane motions of a satellite with a movable mass is written in the following form

$$(A + ml^2)(\ddot\varphi + \ddot\nu) + 2ml\dot l(\dot\varphi + \dot\nu) = 3n^2 k_1^2 k_2^3 (B - A - ml^2)\sin\varphi\cos\varphi; \quad l = l(\varphi, \dot\varphi) \tag{1.5}$$

Treating the true anomaly as a new variable,[3] according to the equality

$$\dot\nu = nk_1 k_2^2 \tag{1.6}$$

we write the first and second derivatives with respect to time for φ in the form

$$\dot\varphi = \varphi' nk_1 k_2^2, \quad \ddot\varphi = n^2 k_1^2 k_2^3 [k_2\varphi'' - 2e\sin\nu\varphi'] \tag{1.7}$$

The prime denotes a derivative with respect to ν. In addition, the following equalities hold

$$\dot l = l' nk_1 k_2^2, \quad \ddot\nu = -2n^2 ek_1^2 k_2^3 \sin\nu \tag{1.8}$$

Finally, when relations (1.6)-(1.8) are taken into account, the equation of plane motions of a satellite with a movable mass in a Keplerian orbit under the action of a gravitational torque is written in the form

$$\begin{aligned} &k_2\varphi'' + 2X\varphi' = 3(B/(A + ml^2) - 1)\sin\varphi\cos\varphi - X \\ &X = 2(k_2 mll'/(A + ml^2) - e\sin\nu) \end{aligned} \tag{1.9}$$

For motion in a circular orbit $e = 0$, and $k_2 = 1$.

2. Choice of the control and equation of controlled motions of a satellite

We will use the principle of action of a swing (a plane pendulum of variable length) to solve the problem of gravitational stability with respect to in-plane perturbations of the relative equilibrium position

$$\varphi = \varphi' = 0 \tag{2.1}$$

of a satellite in a circular orbit. We will treat the distance from the centre of mass of the carrier body O_1 to the movable mass m_2, which is a continuous function of the phase state vector, as a control. By analogy with swings, we can define a law for controlling the movable mass, which will promote a decrease or an increase in the amplitude of the oscillation of the satellite in the vicinity of the relative equilibrium position under the action of the gravitational torque, depending on the values of the coefficients. The continuity of the control law selected enables us to construct Lyapunov functions based on classical stability theory for analytically investigating the asymptotic stability and instability of plane motions of the satellite.

We will state and solve the problem of the asymptotic damping of in-place oscillations of a satellite about relative equilibrium position (2.1). We will obtain the solution by the second method of stability theory.

We define the control in the form

$$l = l_0 + a\varphi'\sin\varphi, \quad a = \text{const} \tag{2.2}$$

Suppose $a > 0$ initially. Taking into account the relation

$$l' = a\varphi''\sin\varphi + a\varphi'^2\cos\varphi$$

we rewrite Eq. (1.9) for $e = 0$ and $k_2 = 1$ in the form

$$W(\varphi, \varphi')\varphi'' = -2F(\varphi' + 1)\varphi'^2\cos\varphi - \frac{3}{2}H\sin 2\varphi \tag{2.3}$$

Here

$$W(\varphi, \varphi') = G + F(3\varphi' + 2)\sin\varphi, \quad F = mal, \quad G = A + ml_0 l, \quad H = A - B + ml^2$$

Equation (2.3) clearly has zero solution (2.1), which corresponds to the relative equilibrium position of the satellite investigated; therefore, it is the equation of perturbed motion in the vicinity of this equilibrium position.

We introduce the notation

$$F_0 = mal_0 > 0, \quad G_0 = A + ml_0^2 > 0, \quad H_0 = A - B + ml_0^2 > 0.$$

We choose the Lyapunov function

$$V = \frac{1}{2}W_0(\varphi,\varphi')\varphi'^2 + \frac{3}{8}(2H_0 + F_0(\varphi' + k)\sin\varphi)(1-\cos 2)$$
$$\approx \frac{1}{2}G_0\varphi'^2 + \frac{3}{2}(A - B + ml_0^2)\varphi^2; \quad W_0(\varphi,\varphi') = G_0 + F_0(3\varphi' + 4)\sin\varphi \tag{2.4}$$

We will determine the coefficient k = const > 0 later. Terms of the third-order and higher in $\dot\varphi$ and φ' were discarded in formula (2.4). As follows from expression (2.4), when the condition

$$W_0(\varphi,\varphi') > \alpha_0 = \text{const} > 0 \tag{2.5}$$

holds, the function $V(\varphi, \varphi')$ in the vicinity of relative equilibrium position (2.1) can be represented by a series that begins with a positive-definite quadratic form. Since the functions are sign-definite,[16] function (2.4) is positive-definite. We will calculate the total derivative of the function $V = V(\varphi, \varphi')$ with respect to time. Since $\dot\nu = n$ in a circular orbit, according to formula (1.6), by virtue of Eq. (2.3) the derivative of the Lyapunov function has the form

$$\dot V = n\frac{W_0(\varphi,\varphi')}{W(\varphi,\varphi')}\left[-2F(\varphi'+1)\varphi'^2\cos\varphi - \frac{3}{2}H\sin 2\varphi\right]\varphi'$$
$$+\frac{3}{8}n\frac{Aml_0}{W(\varphi,\varphi')}\left[-2F(\varphi'+1)\varphi'^2\cos\varphi - \frac{3}{2}H\sin 2\varphi\right](4\varphi'^2 + 1 - \cos 2\varphi)$$
$$+\frac{n}{8}\{F_0(12\varphi'+16)\varphi'^3\cos\varphi + 3F_0(\varphi'+k)\varphi'(1-\cos 2\varphi)\cos\varphi$$
$$+6(2H_0 + F_0(\varphi'+k)\sin\varphi)\varphi'\sin 2\varphi\}$$

Expanding the right-hand side of this expression in series in the variables φ and φ' and discarding terms of higher than the fourth order, we obtain

$$\frac{1}{n}\dot V \approx -\frac{1}{2}F_0\varphi'^4 - 4\frac{F_0^2}{G_0}\varphi\varphi'^3 - \left(\frac{15}{4}F_0 + \frac{15}{2}\frac{F_0}{G_0}H_0\right)\varphi^2\varphi'^2$$
$$-12\frac{F_0^2}{G_0^2}H_0\varphi^3\varphi' - \left(6\frac{F_0}{G_0}H_0 - \frac{9}{4}F_0k\right)\varphi^2\varphi' - \frac{9}{4}\frac{F_0}{G_0}H_0\varphi^4 \tag{2.6}$$

In order for the penultimate third-order term on the right-hand side of equality (2.6) to be identically equal to zero, we choose the coefficient k in the form

$$k = \frac{8}{3}\frac{H_0}{G_0} \tag{2.7}$$

Then derivative (2.6) will be a homogeneous fourth-order form in the variables φ and φ'.

By Sylvester's criterion,[16] when the inequality

$$\frac{15}{8} - 4\left(\frac{F}{G}\right)^2 > 0 \quad \Leftrightarrow \quad \frac{mal_0}{A + ml_0^2} < \frac{\sqrt{30}}{8}$$

holds, which occurs by virtue of the smallness of m, homogeneous form (2.6) will be negative-definite; it corresponds to positive-definite function (2.4). By Lyapunov's asymptotic stability theorem,[16] relative equilibrium position (2.1) of the satellite in a circular orbit is asymptotically stable. Function (2.4) increases as $|\varphi|$ increases for all $\varphi \in [-\pi/2, \pi/2]$. Therefore,[17] when condition (2.5) is satisfied, control (2.2) will damp the in-plane oscillations of the satellite that begin not only in a small vicinity of equilibrium position (2.1), but also for any initial deviations $\varphi(t_0) \in [-\pi/2, \pi/2]$.

The instability of the equilibrium position

$$\varphi = \pi, \quad \varphi' = 0 \tag{2.8}$$

of Eq. (2.3), by virtue of which the region of attraction of the zero solution increases to $\varphi(t_0) \in (-\pi, \pi]$ and values of $\varphi'(t_0)$ that satisfy inequality (2.5), will be demonstrated below. For, values of the speed $\varphi'(t_0)$ as large as desired, by virtue of the energy analysis performed in Ref. 11 for a similar (multi-step) law for controlling the movable mass, we have a decrease in the total energy in a geometric progression, which leads to a decrease in $\varphi'(t)$, particularly down to values that satisfy inequality (2.5). Thus, trivial solution (2.1) is asymptotically stable for any initial deflection. The results of integrating the equations of motion confirm the conclusions drawn.

The upper part of Fig. 2 shows the phase portrait of Eq. (2.3) with control (2.2), which was obtained by numerically integrating the equation of motion for a = 5 m and the following numerical values of the parameters of the system:

$$m = 1\,\text{kg}, \quad A = 100\,\text{kg}\cdot\text{m}^2, \quad B = 10\,\text{kg}\cdot\text{m}^2, \quad l_0 = 10\,\text{m} \tag{2.9}$$

and the initial values

$$\varphi(t_0) = 1\,\text{rad}, \quad \dot\varphi(t_0) = 0.5\,\text{rad/s}$$

The integration was performed in the range $\nu \in [0,100]$ rad. The phase trajectory displays the asymptotic decay of the amplitude and speed of the oscillations of the satellite about the zero equilibrium position, which begin at fairly large values of the initial deflection.

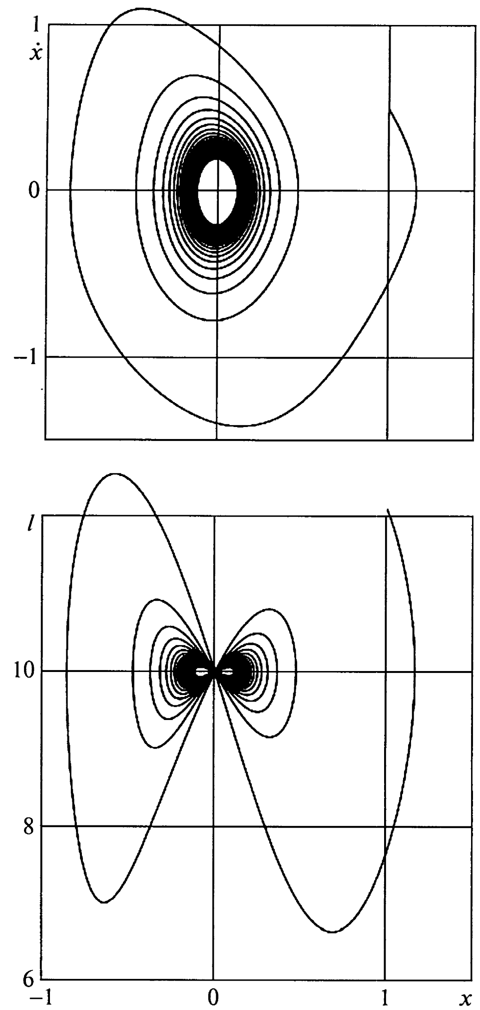

Fig. 2.

The lower part shows the dependence of the distance l on the angle of deflection φ of the satellite, which demonstrates its asymptotic convergences to the value l_0.

3. Swinging and reorientation of the satellite

It is well known[3] that along with the relative equilibrium position in the orbit, at which the axis of symmetry of the satellite is directed along the radius of the local vertical, the satellite also has a diametrically opposite equilibrium position. We apply a control law of the form (2.2) to the problem of the swinging of a satellite from an arbitrary neighbourhood of the relative equilibrium position and its diametrical reorientation. We note that, as in the case of an ordinary pendulum of variable length,[12] when the initial values are

$$\varphi(t_0) = 0, \quad \varphi'(t_0) = 0$$

system (1.9) with $e = 0$ and $k_2 = 1$ is uncontrollable for all $\nu_0 \le \nu < \infty$ under any control law of the form

$$l = l(\varphi, \varphi')$$

However, if we could make equilibrium position (2.1) Lyapunov unstable by adjusting the control law, a controlled swinging of the satellite would become possible when there is a small deviation from this equilibrium position.

We will assume that in control law (2.2) the parameter

$$a = \text{const} < 0 \tag{3.1}$$

The equation of controlled motion of the satellite maintains the form (2.3). The function $V(\varphi, \varphi')$ (2.4) is positive-definite in the vicinity of equilibrium (2.1) when condition (2.5) holds. By analogy with the case considered at the end of Section 2, if we take into account that now

$$F_0 = mal_0 < 0$$

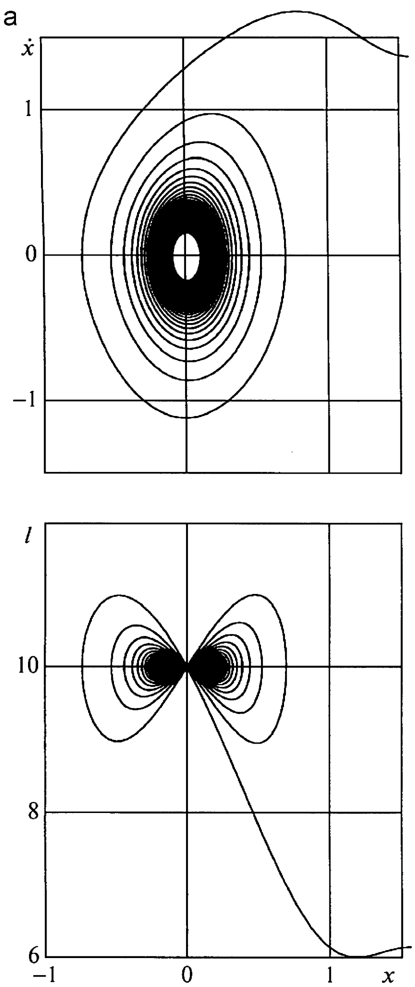

Fig. 3.

we can conclude that when the inequalities

$$\frac{15}{8}-4\left(\frac{F_0}{G_0}\right)^2>0 \;\Leftrightarrow\; \frac{mal_0}{A+ml_0^2}>-\frac{\sqrt{30}}{8}$$

hold, homogeneous form (2.6) with (2.7) will be positive-definite, and will correspond to positive-definite function (2.4). According to Lyapunov's first instability theorem,[16] relative equilibrium position (2.1) of the satellite in a circular orbit is unstable. In addition, by virtue of the increase in the function (2.4) as $|\varphi|$ increases in the set $\varphi\in[-\pi/2, \pi/2]$, any trajectory that begins in the vicinity of equilibrium (2.1) leaves this set. Thus, control (2.2) with a negative value of the parameter a implements the swinging of the satellite about the local vertical.

We will investigate the behaviour of the satellite with control (2.2) for positive and negative values of the parameter a in the vicinity of diametrically opposite equilibrium position (2.8). Introducing the deflection $\varphi=\pi+x$, we write the equation of perturbed motion

$$x''(A+ml(l_0-3ax'\sin x-2a\sin x))=2mla\cos x(x'+1)x'^2-\frac{3}{2}(A+ml^2-B)\sin 2x \tag{3.2}$$

Suppose $a=\text{const}>0$. Then Eq. (3.2) with control (2.2) is identical to Eq. (2.3) with control (2.2) and $a=\text{const}<0$. Therefore, the zero solution $x=x'=0$ of Eq. (3.2) is unstable according to the result obtained in Section 3. Consequently, assuming that the band

$$\varphi\in(-\pi,\pi],\quad \varphi'\in(-\infty,\infty)$$

is the phase space of the system under investigation, we have asymptotic stability, as a whole, of equilibrium position (2.1) of Eq. (2.3).

Now suppose condition (3.1) is satisfied. Equation of perturbed motion (3.2) with control (2.2) and condition (3.1) is identical to Eq. (2.3) with $a=\text{const}>0$. Therefore, the zero solution $x=x'=0$ of Eq. (3.2) is asymptotically stable, by the result obtained in Section 2.

Thus, control (2.2) under condition (3.1) implements the diametrical reorientation of the satellite. After swinging about the relative equilibrium position, at which the axis of dynamic symmetry of the satellite coincides with the local vertical, the satellite swings through an angle π and performs asymptotically decaying oscillations in the vicinity of its opposite position of relative equilibrium in the orbit. This process is clearly illustrated by the graphs of the corresponding numerical calculations: because of the symmetry of Fig. 3a about the $x=\pi/2$ axis and of Fig. 3b about the $x=-\pi/2$ axis, only the left-hand parts of the graphs are shown for Fig. 3a, and only the right-hand parts are shown for Fig. 3b.

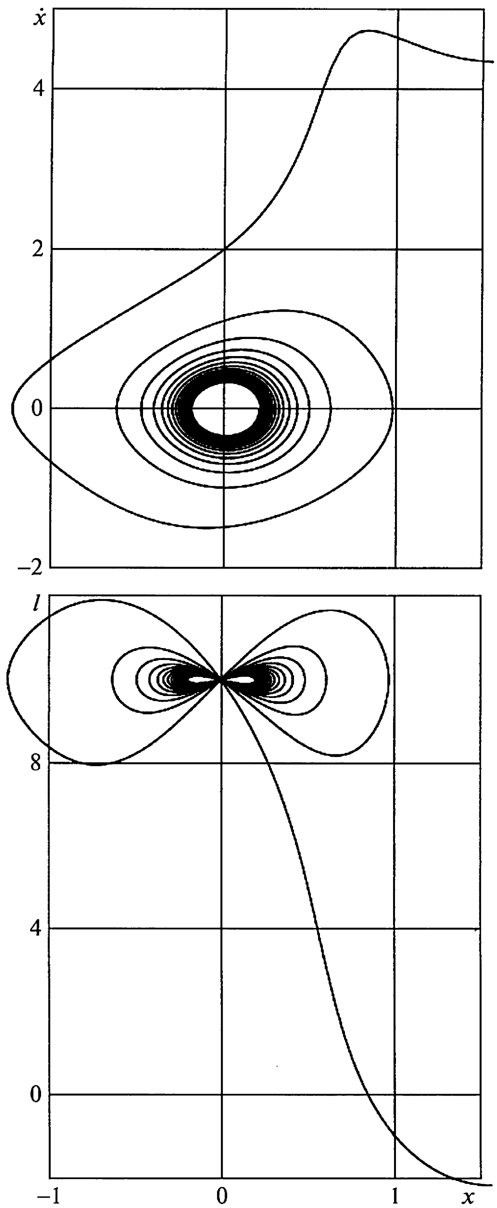

Fig. 4.

Figure 3a (its upper part) shows the phase portrait of Eq. (2.3) under control (2.2) and condition (3.1), which was obtained by numerical integration of the equation of motion in the range $\nu \in [0{,}500\,\text{rad}]$ for $a = -2.8$ m, numerical values (2.9) of the other parameters and the initial data

$$\varphi(t_0) = 0.1\,\text{rad}, \quad \varphi'(t_0) = 0\,\text{rad/s}$$

The phase trajectory reflects the process of swinging about zero equilibrium position (2.1) followed by an asymptotic approach to the new equilibrium position (2.8).

The lower part of Fig. 3a shows the behaviour of the distance l between the centre of mass of the carrier body and the movable mass as a function of the angle φ. Initially, as the satellite swings, the deviations of the distance l from the value l_0 in the vicinity of equilibrium (2.1) increase periodically, and after the turning of the satellite and its transit into the vicinity of position (2.8), the distance l converges asymptotically to l_0. The turning of the satellite is counter-clockwise.

The swinging process and the turning direction during the reorientation of the satellite depend on the values of its initial deviations and the value of the parameter a in control (2.2).

The upper part of Fig. 3b shows the phase portrait of controlled motions (2.3) in the range $\nu \in [0{,}120\,\text{rad}]$ for $a = -3$ m, the same numerical values (2.9) of the parameters of the system and the initial data

$$\varphi(t_0) = 0.1993\,\text{rad}, \quad \varphi'(t_0) = 0\,\text{rad/s}$$

The phase trajectory reflects the process of swinging about zero equilibrium position (2.1) and clockwise turning of the satellite with an asymptotic approach to the new equilibrium position

$$\varphi = -\pi, \quad \varphi' = 0 \tag{3.3}$$

The lower part of Fig. 3b shows the behaviour of the distance l as a function of the angle φ. After periodic increases in the deviation of the value of l from l_0 in the vicinity of equilibrium (2.1) and turning of the satellite, asymptotic convergence of l to l_0 in position (3.3) is observed.

The transitional reorientation process can be controlled by varying the value of the parameter a under the same initial conditions. The upper part of Fig. 4 (again, because of the symmetry of the graphs about the $x=\pi/2$ axis, only their left-hand parts are presented) shows the corresponding phase portrait of the controlled motions for $a=-2.8$, the same values of the other parameters and the same initial conditions as in Fig. 3b and illustrates the turning of the satellite, which is again counter-clockwise, from position (2.1) to position (2.8). The lower part of Fig. 4 shows the corresponding behaviour of l.

Note that the magnitude and the rate of variation $\dot{l}$ of the length l are important characteristics of the transitional process of satellite reorientation. For example, Fig. 4 shows that $l<0$ during the turning, which corresponds to displacement of the movable load along the longitudinal axis beyond the centre of mass of the satellite O_1. During the reorientation which corresponds to the lower part of Fig. 3b, the displacements of the load are significantly less and occur in the vicinity of its initial position l_0. Clearly, by choosing the parameter a of control law (2.2) in accordance with the initial deflection of the satellite $\varphi(t_0)$, the desired turning direction can be obtained, and the magnitude of the largest deviation of the load during reorientation of the satellite can be limited.

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