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CONTROL OF A TETHER DEPLOYMENT SYSTEM FOR DELIVERY OF A RE-ENTRY CAPSULE

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The field of space tethers has received very much attention in recent decades. The central advantage of using tethers in many of these applications is that very little fuel needs to be consumed. The tethered systems offer numerous ways of beneficial implementation on modern spacecrafts and allow to perform multiple tasks including such as payload delivery from the Earth orbit. It is the task of payload delivery from an orbit is the closest to wide practical realization from all other space tether's tasks. As demonstrated by the mission of YES2 a re-entry capsule can be returned to Earth by a tether. Braking of the capsule is achieved using momentum provided from the swinging tether. The more a deflection angle of the tether from the local vertical, the more braking effect of the capsule is achieved.

The goal is to find the control law that allows one to increase the angle of deflection of the tether from the local vertical, i.e., to increase momentum provided from the swinging tether. This control law can be applied to the final phase of the deployment of the tether, both for dynamic so the static deployment. The control law is based on the principle of a swing with variable length. Simulations show that the system can be controlled quite well using the proposed control law for the tether length rate. The effects of orbit eccentricity and viscoelastic properties of a tether were incorporated into the mathematical model to allow more accurate trajectories to be computed. The control method allows to reduce a required tether length for deliver capsules on Earth's surface. Using this method, we have shown that it is possible to diminish tether length at 5 km as compared with YES2 mission. Results of the numerical modeling showed that the control law is effective for the final phase of the tether deployment, when the initial deployment occurs by means static or dynamic scheme.

I. INTRODUCTION

The advent of tethered satellite systems (TSS) starts a new era in space research. The fundamental paper by Beletsky and Levin[1] has played an important role in providing the basis for the study of the tethered system dynamics. The tethered systems offer numerous ways of beneficial implementation on modern spacecrafts and allow performing multiple tasks including such as payload delivery from the Earth orbit[1-7]. The task of payload delivery from an orbit is the closest to wide practical realization from all other space tether's tasks. There are two essentially different approaches to tether deployment at the solution of payload descending problem. They are received a title of static and dynamic deployment[4]. The first way is termed as "static deployment" means slow release of a tether which all time is in a neighbourhood of a local vertical. The second way is named "dynamic deployment". It means a swing of a tether at the expense of Coriolis force acting on it and use of it oscillations for additional decreasing of payload velocity[4]. Some successful experiments of payload delivery by means of tether were executed at present time. In 1993 SEDS-1 mission[5] and in 2007 YES2 mission were made[6,7]. In first one the static deployment was used, and in second - dynamic deployment. Dynamic deployment requires the tether shorter length than under static deployment. On the other hand the dynamic deployment process is very complicated[6,7]. It turns out we can get dignity the dynamic deployment under static deployment if the tether to sway from side to side at the final stage only. It is sufficient to use a simple control law for the length tether in the final stages of deployment, when the tether is fully released, except for a small segment of the tether required for control.

This paper is organized as follows. In Section 1, aim of this paper is formulated. In Section 2, a planar motion of two material points (mother satellite and reentry capsule) connected by an inextensible tether on a circular orbit is considered. Section 3 gives averaged equation and analytical solution of motion of the capsule relative to the mother satellite. In Section 4, the effectiveness of the control law of the tether length is illustrated by numerical simulation.

This study focuses on a control strategy for the final phase of the deployment of the tether system for payload delivery to Earth’s surface, which leads to an increase of a deflection angle of a tether from a local vertical and hence reduces perigee altitude of a re-entry trajectory of a capsule. The control law is based on the principle of a swing with variable rope

$$\dot{l}_0 = -\lambda\dot{\alpha} \qquad [1]$$

where l_0 is total length of the tether, $\lambda > 0$ is a constant coefficient. Similar control laws have been used in the tasks of gravitational stability of a satellite[8] and of a mathematical pendulum[9].

II. MATHEMATICAL MODEL

We consider only planar motion of the tethered system in the orbital plane. The tethered system consists of a mother satellite, the capsule, and a viscoelastic tether between the two (Fig. 1). The mother satellite and the capsule are modeled as material points which have masses m_m and m_c respectively, at that $m_c << m_m$. The mother satellite moves on a circular orbit. The tether is weightless and tether's length l is always much smaller than the mother satellite orbital radius $\left(l << R_0\right)$. Taking into account the accepted assumptions the motion equations of the capsule relative to the mother satellite can be written as

$$\ddot{\alpha} + \ddot{\theta} + 2\frac{\dot{l}}{l}\left(\dot{\alpha} + \omega\right)$$

$$+3\frac{\mu}{R_0^3}\sin\alpha\cos\alpha = 0, \qquad [2]$$

$$m_c\ddot{l} = 2m_c\omega^2 l\cos^2\alpha + m_c\dot{\alpha}^2 l - T, \qquad [3]$$

$$\dot{l}_0 = -\lambda\dot{\alpha} \qquad [4]$$

where $\omega = \sqrt{\mu R_0^{-3}}$, μ is the gravitational constant of the Earth.

Assuming that there is the inextensible tether, the equations of motion [2]-[4] are simplified to a single equation of motion

$$\alpha'' + 3\sin\alpha\cos\alpha - 2\frac{\lambda\alpha'}{l_0}\left(\alpha' + 1\right) = 0 \qquad [5]$$

where $\left(\ \right)' = d\left(\ \right)/d\theta$ is derivative with respect to the true anomaly $\theta = \omega t$. Then the tether tension force is

$$T = m_c\left(\omega^2\left(\lambda\alpha'' + \alpha'^2 l_0\right) + \frac{2g_0 l_0}{R_0}\cos^2\alpha\right) \qquad [6]$$

where g_0 is gravitational acceleration of the mother satellite (Fig.1).

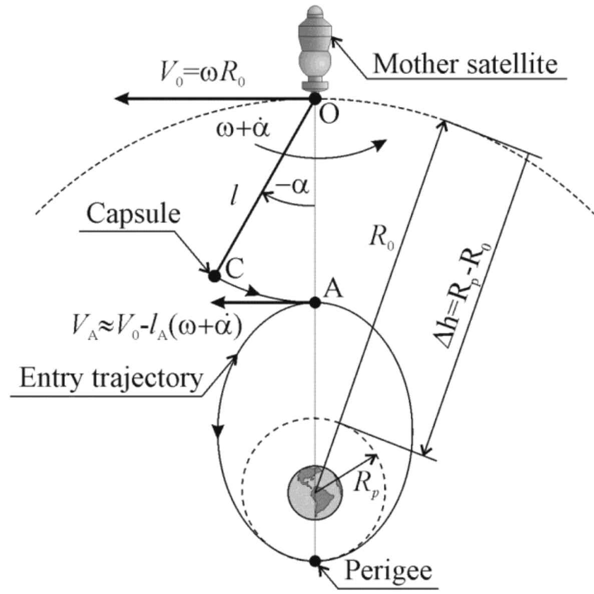

Fig. 1. Swinging release of a capsule from tether

III. AVERAGED EQUATION AND ANALYTICAL SOLUTION

In order to find an approximate solution of the equation [5], we assume that control coefficient λ is always much smaller than the tether length

$$\varepsilon = \lambda / l_0 << 1 \qquad [7]$$

By means of the assumption [7], the equation [5] can be written as

$$\alpha'' + \nu^2\sin\alpha\cos\alpha = 2\varepsilon\left(\alpha' + 1\right)\alpha' \qquad [8]$$

where $\nu^2 = 3$.

The perturbed equation [8] shows that the tether oscillates relative to the position $\alpha = 0$ with a slowly varying amplitude of the deflection angle α_m. If the small parameter equal to zero $\left(\varepsilon = 0 \rightarrow l_0 = const\right)$, then the unperturbed equation takes place

$$\alpha'' + \nu^2\sin\alpha\cos\alpha = 0 \qquad [9]$$

Now we write the energy integral for the equation [9]

$$\frac{\alpha'^2}{2} - \frac{\nu^2}{4}\cos 2\alpha = W \qquad [10]$$

Taking into account the equation [8], we obtain the derivative of the energy integral

$$W' = 2\varepsilon(\alpha'+1)\alpha'^2$$

and averaging of this equation over the period of the variable θ

$$T_\theta = \oint d\theta \qquad [11]$$

we get

$$W' = \frac{2\varepsilon}{T_\theta}\oint(\alpha'+1)\alpha'^2 d\theta \qquad [12]$$

Solving the equation [10] with respect to

$$(\alpha') = \pm\sqrt{2\left(W + \frac{\nu^2}{4}\cos 2\alpha\right)}$$

the equations [12] and [11] can be written as

$$W' = \frac{8\varepsilon}{T_\theta}\int_0^{\alpha_m}\sqrt{2\left(W + \frac{\nu^2}{4}\cos 2\alpha\right)}d\alpha,$$

$$T_\theta = \oint d\theta = 4\int_0^{\alpha_m}\frac{d\alpha}{\sqrt{2\left(W + \frac{\nu^2}{4}\cos 2\alpha\right)}}$$

The integrals in the right-hand sides of these equations are elliptic integrals. Change of variable

$$\sin\alpha = k\sin\varphi \qquad (k = \sin\alpha_m)$$

converts these integrals to the complete elliptic integrals of the first and second kind[10]

$$K(k) = \int_0^{\frac{\pi}{2}}\frac{d\varphi}{\sqrt{(1+k^2\sin^2\varphi)}},$$

$$E(k) = \int_0^{\frac{\pi}{2}}\sqrt{(1+k^2\sin^2\varphi)}d\varphi$$

This finally leads to the following equations

$$W' = 2\varepsilon\nu^2\left[\frac{E(k)}{K(k)} - (1-k^2)\right],$$

$$T_\theta = \frac{4}{\nu}K(k) \qquad [13]$$

From the equation [10] it follows that

$$W(\alpha,\dot\alpha) = W(\alpha_m, \dot\alpha = 0) = 2\sin^2\alpha_m - 1$$
$$= 2k^2 - 1 = 2x - 1 \qquad [14]$$

where $x = k^2 = \sin^2\alpha_m$ is a new variable.
The variable substitution [14] in the equation [13] gives

$$x' = \frac{8\varepsilon}{\Omega}\left[\frac{E(\sqrt{x})}{K(\sqrt{x})} - (1-x)\right]$$

This equation is approximated by a cubic polynomial

$$\frac{dx}{d\theta} = -\frac{\varepsilon}{8}x\left(x^2 + 2x + 16\right) \qquad [15]$$

Separating the variables in [15] and integrating it, we get

$$4a\varepsilon(\theta-\theta_0) = \Big[\left(\sqrt{a}-a\right)\ln\left(\sqrt{a}+1+x\right)$$
$$-\left(\sqrt{a}+a\right)\ln\left(\sqrt{a}-1-x\right) + 2\ln(x)\Big]_{\sin^2\alpha_{m0}}^{\sin^2\alpha_m} \qquad [16]$$

where $a = 17,\ \alpha_{m0} = \alpha_m(\theta_0)$

IV. THE EFFECTIVENESS OF THE CONTROL LAW

To evaluate the effectiveness of the control law [1] we use the formula determining a change in altitude in the perigee of the capsule after the separation from the tether in the point A (Fig.1). It is given by[1]

$$\Delta h = R_p - R_0 = \frac{(R_A V_A)^2}{2\mu - R_A V_A^2} - R_0$$

where R_p is a perigee height of a re-entry trajectory, $R_A = R_0 - l_A,\ V_A \approx \omega\left[R_0 - l_A(\alpha'_A + 1)\right]$, as shown in Fig. 1. We note that for the YES2 mission the change in altitude was

$$\Delta h_{YES2} \approx -330\,km$$

when the amplitude $\alpha_{\mathrm{m}} = 40\deg$, the tether length

$l = 30\,km$ and $R_0 = 6645\,km$ [7]

To illustrate the performance of the proposed method, consider an application of the law [1] and compare with the mission YES2 mission[7]. The simulation results are depicted in Figs. 2-4. Fig. 2 shows the true anomaly history of the tether libration angle α. We see that after 5 complete librations the value of the librations the amplitude of the tether librations is almost twice the value of the angle (40 deg) and reaches 74.5 deg. After which the capsule is separated from the tether when the tether reaches to the local vertical. During this time the Mother satellite makes approximately 4.2 revolutions around the Earth. Fig. 3a depicts the true anomaly history of the tether length. We note that the tether length lies in the range from 24 km to 25.3 km. Fig. 3b shows that the tether tension less than 2 N and the tether always remains stretched. Note that the simulation is performed for $\lambda = 750\,m$.

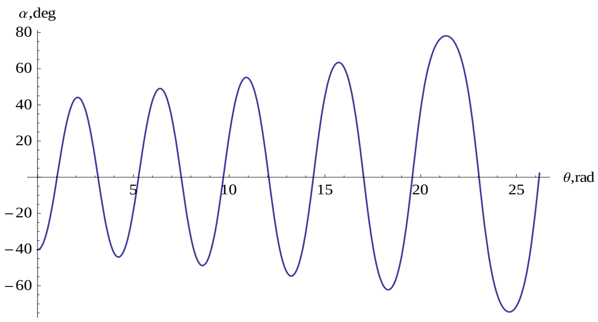

Fig. 2. True anomaly history of the tether libration angle

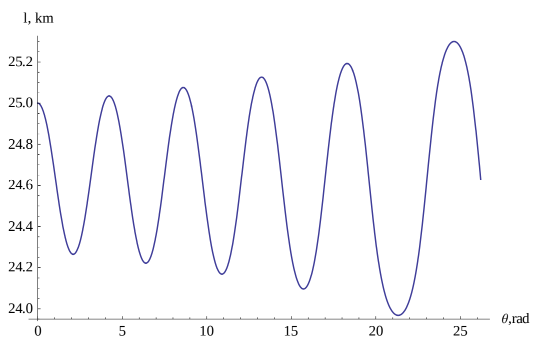

Fig. 3. True anomaly histories of the tether length

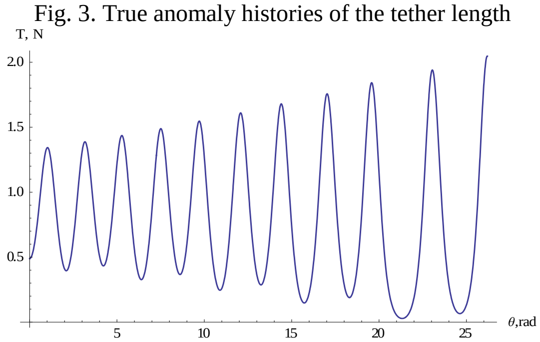

Fig.4. True anomaly histories of the tether tension

For this numerical experiment the change in altitude equals

$$\Delta h = -335km$$

V. CONCLUSION

It is important to remember that all artwork, captions, figures, graphs and tables will be reproduced exactly The principle of variable-length swings in the final phase of the tether deployment has been allowed to reach any value of the amplitude of the tether oscillation of the range $a_m \in \left(0, \pi / 2\right)$. The obtained analytical solution shows the relationship between the parameters of the tether system, the control law and the amplitude of the tether oscillation α_m. The control method allows reducing a required tether length for deliver capsules on Earth's surface. Using this method, we have shown that it is possible to diminish tether length at 5 km as compared with YES2 mission. Results of the numerical modeling showed that the control law is effective for the final phase of the tether deployment.

VI. ACKNOWLEDGMENTS

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REFERENCES

[1] V.V. Beletsky, E.M. Levin, *Dynamics of space tether systems*, Univelt, Univelt Incorporated, 1993. –T. 83.

[2] E.M. Levin, *Dynamic analysis of space tether missions*, Univelt Incorporated, 2007. -T. 126.

[3] V.S. Aslanov, A.S. Ledkov, *Dynamics of the Tethered Satellite Systems*, Elsevier, 2012.

[4] F. Zimmermann, U. M. Schoettle, E. Messerschmid, “Optimization of the tether-assisted return mission of a guided re-entry capsule”, Aerospace Science and Technology 9, 2005, pp. 713–721.

[5] L. Johnson, R.D. Estes, E. Lorenzini, M. Martinez-Sanchez, Propulsive Small Expendable Deployer System Experiment, J. Sanmartin, Journal of Spacecraft and Rockets 37, 2, 2000, pp. 173–176.

[6] M. Kruijff, *Tethers in Space*, Ph.D. Thesis, Delft University of Technology, The Netherlands, 2011.

[7] P. Williams, A. Hyslop, M. Stelzer, M. Kruijff, YES2 optimal trajectories in presence of eccentricity and aerodynamic drag, Acta Astronautica 64, 2009, pp. 745–769.

[8] V.S. Aslanov, S.P. Bezglasnyi, Journal of Applied Mathematics and Mechanics, 76, 4, 2012, pp. 405-412.
[9] V.S. Aslanov, S.P. Bezglasnyi, Mechanics of Solids, 47, 3, 2012, pp. 285-297.
[10] G. Korn, T. Korn, “Mathematical handbook”, McGraw-Hill Book Company, New York, 1968.