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TETHER LENGTH CONTROL IN TETHER-ASSISTED DEORBITING MISSION FROM AN ELLIPTICAL ORBIT

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Abstract

Among proposed applications of tethers in space, tether-assisted payload deorbiting projects are closest to the wide practical implementation. Three experiments were carried out successfully to date. The idea is to avoid the use of jet engines to generate a brake pulse, which transfers the payload on the descent orbit. The necessary reduction in velocity can be obtained through the use of features of the orbital motion of the space tether systems. Development of effective tether control methods are of great scientific and technological interest. The majority of research concerns the descent of a payload from a circular orbit. The aim of this research is development of an effective method of the tether control in tether-assisted payload deorbiting mission in the case of elliptical orbit of small eccentricity.

The plane motion of the space tether system, which consists of a satellite, massless tether and payload, is considered. It is supposed that the center of mass of the system moves on a Keplerian orbit with small eccentricity. The tether length control law, which is based on a swing principle, is proposed. The equation of the controlled relative motion of the tether around the center of mass is obtained. The angle of the true anomaly is used as an independent variable. It is shown that in the vicinity of the local vertical a stable limit cycle with period 2 pi may exists for certain values of the coefficient in the control law. An approximate analytical equation of this cycle is obtained. It is shown that considered control law can transfer the system into rotation mode, which is more preferable for the tasks of payload deorbit, as it provides a greater reduction in the payload velocity. The optimal value of the control coefficient and the moment of payload separation, which ensure transfer of the payload into the orbit with a minimum altitude of the perigee, are founded numerically for the case of small eccentricity. The proposed control law can be used to develop new deorbiting systems.

Keywords: space tether, payload return, elliptical orbit, limit cycle, control

1. Introduction

Space tethered systems have attracted the attention of researchers since the beginning of the space age. Their large extent, variable configuration, and the possibility of interaction with the Earth electromagnetic field create prerequisites for the wide use of space tethers for solving various transport tasks in space. Detailed reviews of various projects and ways of space tethered systems application are given in the works [1-3]. Among proposed applications of tethers in space, tether-assisted payload deorbiting projects are closest to the wide practical implementation. Three experiments were carried out successfully to date: SEDS-1 in 1993, SEDS-2 in 1994, and YES2 in 2007 [4]. These experiments proved the possibility of removing a payload from orbit by space tether without the use of jet fuel. Delivery of a payload to the Earth surface is the final stage of a large number of space missions, thus the development of cheap and environmentally friendly technology of payload deorbit is relevant.

Any mission of payload deorbit can be divided into three stages: braking to transfer from the initial orbit to breaking orbit, flight to the boundary of the atmosphere, descent into the atmosphere. Usually the transfer from the initial orbit is carried out by jet engines. However, this stage can be realized by the means of a space tether. There are two main tether length control schemes that are actively discussed in the scientific literature and can be used for the tether-assisted payload descent: “static” and “dynamic” [5, 6]. The static scheme implies the slow deployment of the tether along the local vertical. The transfer of the payload to the braking orbit is due to the fact that the payload descends to a point where its velocity after separation from the tether is insufficient for existence in a near-earth orbit. The dynamic scheme is based on the use of the Coriolis force to deflect the tether from the local vertical in the direction of the orbital flight. The absolute velocity of the payload is reduced by the subsequent tether return oscillation. Like a mathematical pendulum, the greater the maximum tether deviation angle from the local vertical, the greater reduction in the payload absolute velocity can be achieved. To increase the tether oscillations amplitude, the swing principle can be applied [7].

The majority of research devoted to the tether-assisted return missions concerns the descent of a payload from a circular orbit [5-9]. This study focuses on tethered return from an elliptical orbit of small eccentricity. In contrast to the case of a circular orbit, the vertical position is not an equilibrium state in the case of elliptical orbit. Moreover, with the growth of eccentricity, the motion of the space tethered system becomes chaotic [10]. Another difference from the case of a circular orbit is that the separation of the payload when it passes the local vertical does not guarantee its transfer into orbit with a minimum radius of perigee. An additional effort should be made to determine the optimum separation moment [11].

The aim of this research is development of an effective method of the tether control in tether-assisted payload deorbiting mission in the case of elliptical orbit of small eccentricity. As a criterion of optimality, the radius of the perigee of the orbit, into which the payload passes after separation from the tether, is used.

2. Mathematical models and methods

2.1 Equations of the motion

Considered mechanical system consists of a satellite, tether and payload. The satellite is equipped with a winch that is capable of unwinding and winding the tether according to a predefined law. The satellite and payload are material points. The centre of mass of the systems moves along the unperturbed Keplerian orbit. Its position is expressed through the distance between the centre of Earth and the satellite r and the true anomaly angle θ (Fig. 1). The motion of the considered system can be described by the equations [7]

$$\alpha'' = 2(\alpha'+1)\left(\frac{e\sin\theta}{1+e\cos\theta} - \frac{l'}{l}\right) - \frac{3\sin\alpha\cos\alpha}{1+e\cos\theta}, \quad (1)$$

$$r = \frac{p}{1+e\cos\theta}, \qquad \dot{\theta} = \sqrt{\frac{\mu}{p^3}}(1+e\cos\theta)^2. \quad (2)$$

Here $(\,)'$ is the derivative with respect to θ, α is the angle between the tether and local vertical (Fig. 1), l is the tether length, e is the eccentricity, p is the orbital parameter, μ is the gravitational constant of the Earth. It is assumed that the tether length changes in accordance with the control law

$$l = -\lambda\alpha + L_0, \quad (3)$$

where λ is a constant control coefficient, L_0 is the tether length for $\alpha = 0$. It is supposed that $\lambda << L_0$ and small parameter ε can be introduced as

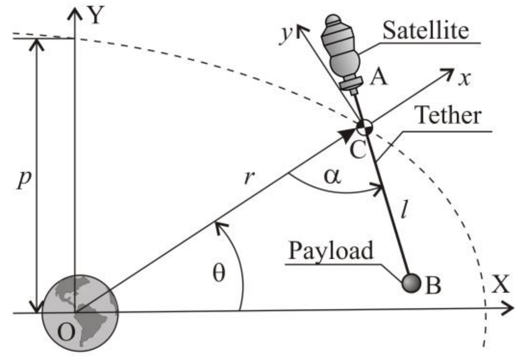

Fig. 1. Space tethered system

$$\varepsilon = \frac{\lambda}{L_0}. \quad (4)$$

The right side of equation (1) contains the term $2(\alpha'+1)l'l^{-1}$ which can be expanded into a series with respect to the small parameter ε. After retaining the term of the order ε, equation (1) takes form

$$\alpha'' = 2(\alpha'+1)\left(\frac{e\sin\theta}{1+e\cos\theta} + \varepsilon\alpha'\right) - \frac{3\sin\alpha\cos\alpha}{1+e\cos\theta}. \quad (5)$$

In the case of small eccentricity, linearization of equation (5) by e yields

$$\begin{aligned}\alpha'' + 3\sin\alpha\cos\alpha &= 2\varepsilon\alpha'(\alpha'+1) \\ &+ e(3\sin\alpha\cos\alpha\cos\theta + 2(\alpha'+1)\sin\theta).\end{aligned} \quad (6)$$

Equation (6) can be expanded in a power series around the point $\alpha = 0$

$$\begin{aligned}\alpha'' + 3\alpha - 2\alpha^3 &= 2eh\alpha'(\alpha'+1) \\ &+ e((3\alpha - 2\alpha^3)\cos\theta + 2(\alpha'+1)\sin\theta).\end{aligned} \quad (7)$$

Here $h = \varepsilon e^{-1}$. Equation (7) is the perturbed Duffing equation. The first term on the right side of (7) is the small disturbance caused by the controlled tether length change. The second term is the disturbance due to the small ellipticity of the orbit.

It should be noted that some constrains should be performed during controlled motion. Firstly, the tether length is limited

$$L_{\min} < l < L_{\max}. \quad (8)$$

Secondly, the rate of the tether deployment/retrieval should not exceed the allowable value

$$|\dot{l}| \leq V_{\max}. \quad (9)$$

2.2 Nature of the tether length control

Proposed control (3) is based on the swing principle. Let us consider its physical nature. The differentiation of the equation (3) gives

$$l' = -\lambda\alpha'. \quad (10)$$

Let us suppose that coefficient λ is positive. According to (3) and (10) the tether length increases when the angle α decreases (the tether rotates clockwise Fig. 2a), and the tether length decreases when the angle α

increases (the tether rotates counter-clockwise Fig. 2b). The motion of the payload can be considered in the rotating Hill coordinate frame *Cxy* (Fig. 1). Moving in the rotating coordinate frame, the payload experiences the influence of the Coriolis force $\mathbf{F}_C = -2m\dot{\boldsymbol{\theta}}\times\mathbf{V}_r$ (Fig. 2). Here m is the payload mass, $\mathbf{V}_r$ is the relative velocity of the payload. The use of the control law (3) leads to the fact that the direction of the Coriolis force coincides with the direction of the tether rotation (Fig. 2). Thus, this force creates a torque relative point C tending to increase the module of α. In the case of a negative control coefficient λ, the Coriolis force creates a torque tending to reduce the amplitude of α angle oscillations.

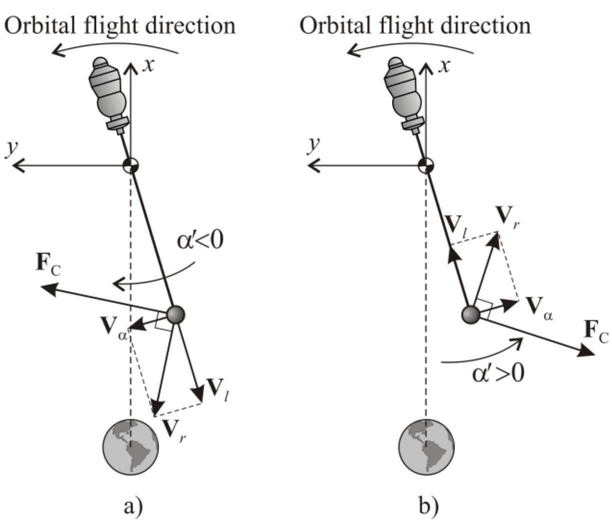

Fig. 2. Influence of Coriolis force

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2.3 Limit cycle

Numerical studies have shown that, there is a stable limit cycle of 2π period near the point $\alpha = 0$, $\alpha' = 0$ when $\lambda > 0$. A limit cycle is an isolated closed trajectory. Neighbouring trajectories spiral toward stable limit cycle. Figure 3 demonstrates stable limit cycles for $\varepsilon = -0.06$ and various eccentricity values. Figure 4 shows the curves $\alpha(\theta)$ obtained for different initial conditions and $e = 0.1$. It can be seen that the curves approach the limit cycle.

The Poincare's method can be used for obtaining periodic analytical solution of equation (7). To find 2π-periodic limit cycle let us write solution $\alpha(\theta)$ in the form of a power series of the small parameter

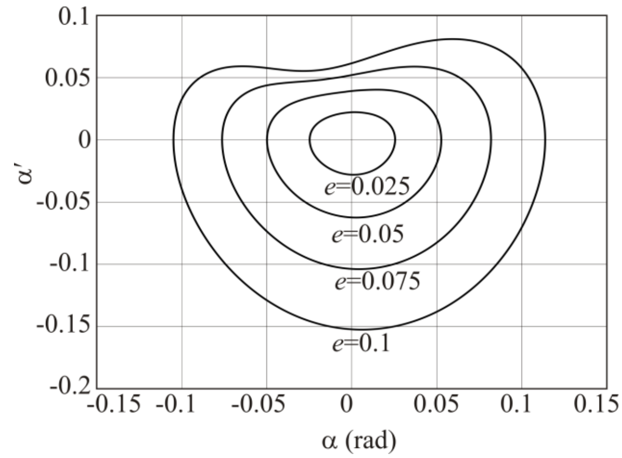

Fig. 3. Limit cycles for various eccentricity values

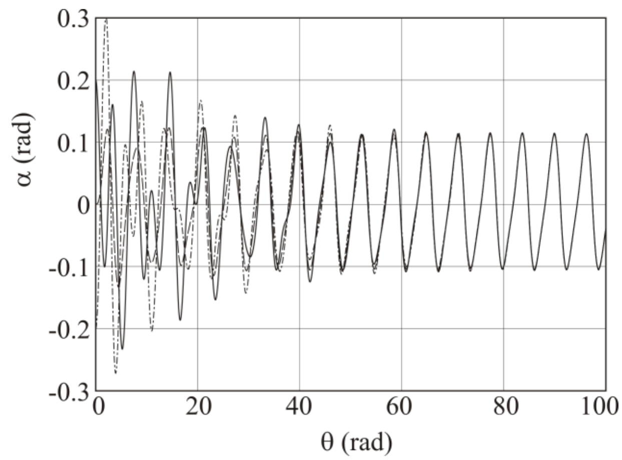

Fig. 4. Tether deflection angle evolution ($e = 0.1$, $\varepsilon = -0.06$)

$$\alpha = \alpha_0(\theta) + e\alpha_1(\theta) + e^2\alpha_2(\theta) + \ldots \qquad (11)$$

where $\alpha_i(\theta)$ are some 2π- periodic functions. After substitution solution (11) into (7) and equating terms of the same powers of e in the left and right side of the result, the system of equations can be obtained. The first equation on this system

$$\alpha_0'' = 2\alpha_0^3 - 3\alpha_0$$

has trivial 2π-periodical solution $\alpha_0 = 0$. Using this solution other equations of the system can be written in rather compact form

$$\alpha_1'' = -3\alpha_1 + 2\sin\theta,$$

$$\alpha_2'' = -3\alpha_2 + 2h\alpha_1' + 2\alpha_1'\sin\theta + 3\alpha_1\cos\theta,$$

$$\alpha_3'' = -3\alpha_3 + 2\alpha_1^3 + 2h(\alpha_1'^2 + \alpha_2') + 2\alpha_2'\sin\theta + 3\alpha_2\cos\theta,$$

$$\alpha_4'' = -3\alpha_4 + 6\alpha_1^2\alpha_2 + 2h(2\alpha_1'\alpha_2' + \alpha_3') + 2\alpha_3'\sin\theta + (3\alpha_3 - 2\alpha_1^3)\cos\theta,$$

$$
\begin{aligned}
\alpha_5'' &= -3\alpha_5 + 6\alpha_1\alpha_2^2 + 6\alpha_1^2\alpha_3 + 2h(\alpha_2'^2 + 2\alpha_1'\alpha_3' + \alpha_4') \\
&\quad + 2\alpha_4'\sin\theta + (3\alpha_4 - 6\alpha_1^2\alpha_2)\cos\theta, \\
\alpha_6'' &= -3\alpha_6 + 6\alpha_1^2\alpha_4 + 12\alpha_1\alpha_2\alpha_3 \\
&\quad + 2h(2\alpha_1'\alpha_4' + 2\alpha_2'\alpha_3' + \alpha_5') + 2\alpha_2^3 + 2\alpha_5'\sin\theta \\
&\quad + (3\alpha_5 - 6\alpha_1^2\alpha_3 - 6\alpha_1\alpha_2^2)\cos\theta, \\
&\ldots
\end{aligned}
\tag{12}
$$

Equations (12) can be solved sequentially. The unknown integration constants should be selected so that the periods of solutions are equal to 2π.

$$
\begin{aligned}
&\alpha_1 = \sin\theta,\ \alpha_2 = h\cos\theta - \frac{5}{2}\sin 2\theta, \\
&\alpha_3 = \frac{h}{2} + \left(\frac{11}{8} - h^2\right)\sin\theta + \frac{13}{2}h\cos 2\theta + \frac{37}{24}\sin 3\theta, \\
&\alpha_4 = -\left(\frac{15}{4}h + h^3\right)\cos\theta + \left(\frac{61}{2}h^2 + \frac{55}{8}\right)\sin 2\theta \\
&-\frac{41}{12}h\cos 3\theta - \frac{175}{208}\sin 4\theta, \\
&\alpha_5 = \left(\frac{25}{8}h - \frac{1}{2}h^3\right) + \left(\frac{367}{32} - \frac{187}{8}h^2 + h^4\right)\sin\theta \\
&-\left(\frac{213}{4}h + \frac{233}{2}h^3\right)\cos 2\theta - \left(\frac{16631}{2496} + \frac{457}{24}h^2\right)\sin 3\theta \\
&-\frac{1107}{1352}h\cos 4\theta + \frac{7567}{9152}\sin 5\theta, \\
&\alpha_6 = \left(\frac{843}{32}h + \frac{165}{2}h^3 + h^5\right)\cos\theta + \left(\frac{20555}{1664}\right. \\
&\left.-\frac{2557}{8}h^2 - \frac{945}{2}h^4\right)\sin 2\theta + \left(\frac{1538777}{32448}h\right. \\
&\left.+\frac{137}{2}h^3\right)\cos 3\theta + \left(\frac{26935}{7436} - \frac{62497}{35152}h^2\right)\sin 4\theta \\
&+\frac{488751}{1308736}h\cos 5\theta - \frac{583075}{604032}\sin 6\theta, \\
&\ldots
\end{aligned}
\tag{13}
$$

The required accuracy of the analytical solution (11) can be achieved by increasing the number of terms in this expansion. Figure 5 shows the results of numerical integration and approximate analytical solutions that take into account the different number of terms in the expansion. The following designation is used for the approximate analytical solutions

$$\alpha^k(\theta) = \sum_{i=0}^{k} e^i \alpha_i(\theta).$$

Graphs of the dependences of the deviations of approximate analytical solutions from the numerical solution

$$\delta_k = \alpha - \alpha^k$$

are shown in Fig. 6. Numerical studies have shown that the limit cycle (11) is stable when $\lambda < 0$, and it is unstable when $\lambda > 0$.

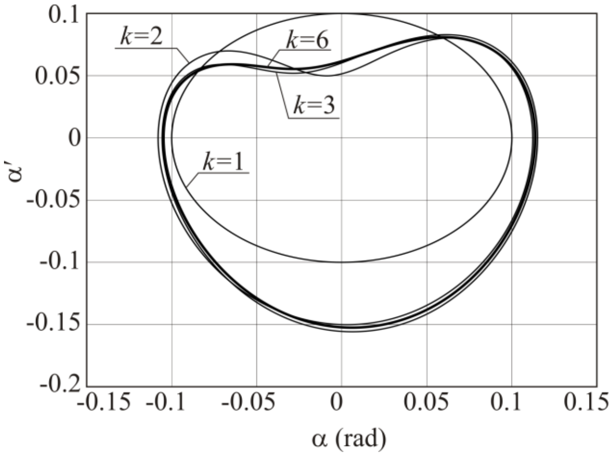

Fig. 5. Comparison of approximate analytical solutions ($e = 0.1$, $\varepsilon = -0.06$)

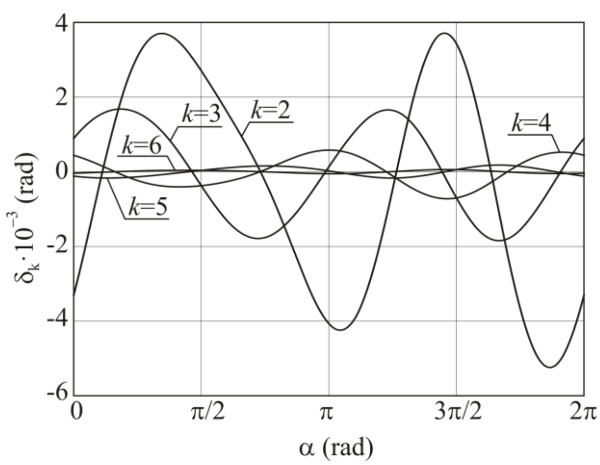

Fig. 6. Deviations of approximate analytical solutions from the numerical solution ($e = 0.1$, $\varepsilon = -0.06$)

2.4 Selection scheme of optimal control parameters

When using the control law (3), three parameters should be defined: control coefficient λ, the moment of the control beginning θ_0, and the moment of the payload separation from the tether θ_s. It is assumed that until the moment θ_0 the motion occurs over a stable limit cycle, then the control coefficient λ reverses the sign, and the swinging the tether starts. As a criterion of optimality, the radius of the perigee of the orbit, into which the payload passes after separation from the tether, is used [12]

$$r_\pi = \frac{H^2}{\mu + f}, \tag{14}$$

where $h = -\dot{x}_B y_B + \dot{y}_B x_B$ is the massless angular momentum, $f_2 = \mu^2 + h^2 c$ is the module of the Laplace-Runge-Lenz vector, $c = \dot{x}_B^2 + \dot{y}_B^2 - \frac{2\mu}{\sqrt{x_B^2 + y_B^2}}$ is the total energy per unit mass, x_B and y_B are the coordinates of the point B in the inertial coordinate frame OXY (Fig. 1)

$$x_B = r\cos\theta - m_A l\cos(\alpha+\theta)(m_A + m_B)^{-1},$$

$$y_B = r\sin\theta - m_A l\sin(\alpha+\theta)(m_A + m_B)^{-1}.$$

Here m_A and m_B are the masses of the satellite and the payload.

The search for optimal control parameters is a difficult task, because the function $r_\pi(\lambda, \theta_0, \theta_s)$ contains a large number of local minima. To find the optimal values the series of numerical simulations of equations (1), (2) with random values of λ and θ_0 should be conducted. The values of the angle θ_s, for which at least one of the constraints (8), (9) is not satisfied, are excluded from consideration. Then the smallest r_π value should be chosen from the set of local minima. The calculation of the controlled motion of the system was performed in Matlab.

3. Results of numerical simulations

As an example, YES2 mission that was studied in detail in [7, 14] will be considered. It is supposed that the center of mass of the space tether system moves in an elliptical orbit with following parameters: the eccentricity is $e = 0.0027$, the perigee attitude is 249 km, the appogee attitude is 285 km. The masses of the satellite and the payload are $m_A = 6530\text{kg}$, $m_B = 12\text{kg}$. It is assumed that the tether length lies within $L_{\min} = 0.5\text{km}$ and $L_{\max} = 31\text{km}$, and the maximum rate of the tether deployment/retrieval is $V_{\max} = 15\text{m/s}$. In [14] it was shown that for a tether of 31km length application of dynamic deployment allows to reduce the payload perigee height by 330 km. The swing principle makes it possible to achieve the same reduction in altitude with a tether length of 25.3 km [7]. Let us show that considered control law (3) can transfer the system into rotation mode, which is more preferable, as it provides a greater reduction in the payload velocity and allows achieving the same reduction in payload perigee altitude with a tether of smaller length. We will use described in section 2.4 scheme and step-by-step method to find optimal control parameters. It is supposed that initially the space tether system moves on the limit cycle (11) with control coefficient $\lambda < 0$.

Numerical calculations have shown that required decrease in perigee height of 330 km can be achieved with a tether of 12.254 km length. This length is 60.47% smaller than in the case of the dynamic YES2 law [14]. The following control parameters were used: $\lambda = 449.225\text{m}$, $\theta_0 = 2\cdot10^{-4}$, $\theta_s = 229.14$. Swinging the tether and the subsequent transition into rotation are observed. The system manages to make three turns around its center of mass until the moment of the payload separation (Fig 7). Fig. 8 shows the dependence of the perigee radius of the orbit on which the payload passes in the case of separation from the tether at the moment θ_s. Fig. 9 demonstrates the change in the tether length during control. Points A-D correspond to local minima in Fig. 8.

The results of the numerical calculation show that the swing principle allows to transfer the space tethered system into rotation mode in the case of an orbit with a small eccentricity. The payload separation from the rotational mode proves to be more effective for the problem of the payload descent than in the case of separation from the oscillation mode. This result agrees well with the known works on moment-exchange tethers [1-4].

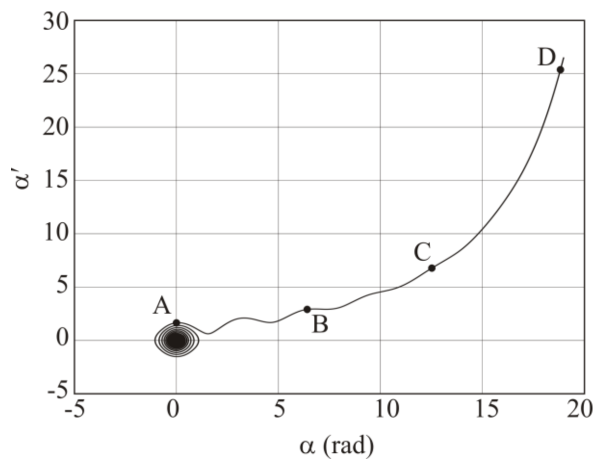

Fig. 7. Phase portrait of equation (1) with control (3)

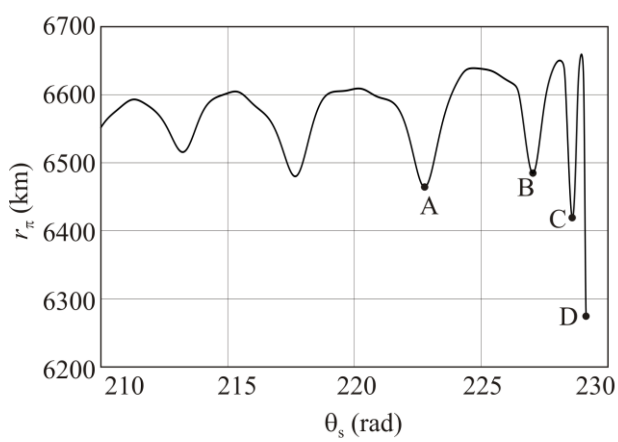

Fig. 8. Dependence of the perigee radius on θ_s

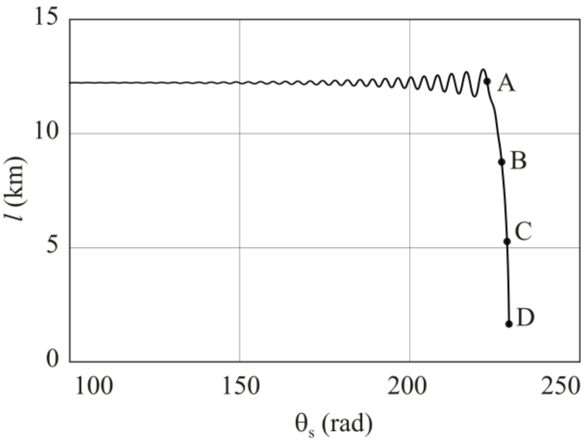

Fig. 9. Dependence of the tether length on θ_s

It should be noted that the payload orbit perigee radius (12) is one of the possible optimality criterion. For real mission planning the set of thermal and mechanical constraints for the re-entry capsule should be taken into consideration [11], but in order to show the effectiveness of the considered control scheme, criterion (12) is sufficient.

4. Conclusions

The plane motion of the space tether system has been considered in this study. The tether length control law, which is based on a swing principle, was proposed. It was shown that in the vicinity of the local vertical a stable limit cycle with period 2π may exists for certain values of the coefficient in the control law. An approximate analytical equation of this cycle was obtained. It was shown that considered control law can transfer the system into rotation mode, which is more preferable for the tasks of payload deorbit, as it provides a greater reduction in the payload velocity. The optimal value of the control coefficient and the moment of payload separation, which ensure transfer of the payload into the orbit with a minimum altitude of the perigee, was founded numerically for the case of small eccentricity. The proposed control law can be used to develop new deorbiting systems.

Acknowledgements

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