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Chaotic motions of tethered tug-debris system with fuel residuals

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Abstract

We consider tethered tug-debris system with a low-thrust tug that is used for the active debris removal. The debris is considered as a rigid body with fuel residuals. The chaotic motion of the system could be caused by the oscillations of the debris object relative to the tether, oscillations of the fuel residuals in the fuel tanks of the debris, eccentricity of the orbit and longitudinal oscillations of the tether. In this paper the chaotic motion induced by the oscillations of the debris object with the fuel residuals relative to the tether is considered. Stable and unstable stationary solutions are presented for the motion of the system in a circular orbit, which depend on the value of the tug's thrust. It is shown that the unstable solutions give rise to the chaotic motion of the system. Poincare sections and Lyapunov exponents are used to detect the chaotic processes in the considered dynamical system.

Keywords: space debris, space tug, tether, chaos

Nomenclature

- m_1 – mass of the space tug;
- m_2 – mass of the space debris;
- m_3 – mass of the fuel residuals;
- l – tether length;
- a – distance from the tether attachment point to the debris center of mass;
- b – distance from the tether attachment point to the fuel pendulum attachment point;
- c – distance from the fuel pendulum attachment point to the center of mass of the fuel residuals
- α – angle between the local vertical axis and the tether
- φ – angle between the tether and the debris axis
- β – angle between the debris axis and the fuel pendulum
- R – distance from the Earth center to the center of mass of the system
- θ – true anomaly angle
- J_x – moment of inertia of the debris relative to the longitudinal axis
- J_z – moment of inertia of the debris relative to the transverse axis

1. Introduction

Space tethers is considered as one of the methods for safe transportation of large space debris objects using space tugs [1–4]. Understanding the dynamical behavior of the tethered tug-debris system is essential for the success of active debris removal missions.

If the space tug and debris connected by a tether move around the Earth in elliptical or circular orbit, the dynamic of the system is caused primarily by the tug's thrust and gravitational force and torque. The eccentricity of the orbit, longitudinal oscillations of the tether cause perturbations in the motion of the system. Tethered tug-debris system can undergo chaotic behavior. The chaotic motion of a tethered system was reported in a paper by Misra et al. [5,6] and by Aslanov [7]. In [5] Misra show that chaos could occur only for elliptic orbits or in the presence of both pitch and roll motion of the system. In [7], it is shown that a chaotic planar motion exists due to the flexibility of the tether and the presence of the low thrust.

Yet another source of the perturbations is the motion of fuel residuals in the tanks of the debris and the motion of the debris as a rigid body relative to the tether. In this paper we investigate the chaotic motion of the system induced by the oscillation of the debris body and the fuel residuals relative to the tether.

In the part 2 the mathematical model of the system described. In the part 3 the chaotic motions of the system are investigated using Poincare sections and Lyapunov exponents.

2. Mathematical model

2.1 Motion of the dug-debris system with fuel residuals

Considered tug-debris system is presented in Fig. 1. The system consists of the space tug, tether, space debris object and the fuel residuals. It is supposed that the space tug has an attitude control system that maintains required orientation of the tug, so the space tug is represented as a point mass C_1. The tether C_1A is considered as a massless rod. The debris is considered as a rigid body with a fuel sloshing mass. C_2 is the center of mass of the debris object. We use the simplest model where the sloshing liquid is modelled as an equivalent pendulum model. This model can be used when the oscillations of liquid are small [8,9]. The pendulum that represents the fuel residuals in the debris fuel tank is represented as the point mass C_3 with mass m_3 attached to the massless rod BC_3 of length c. The rod BC_3 attached to the debris at the point B on its longitudinal axis.

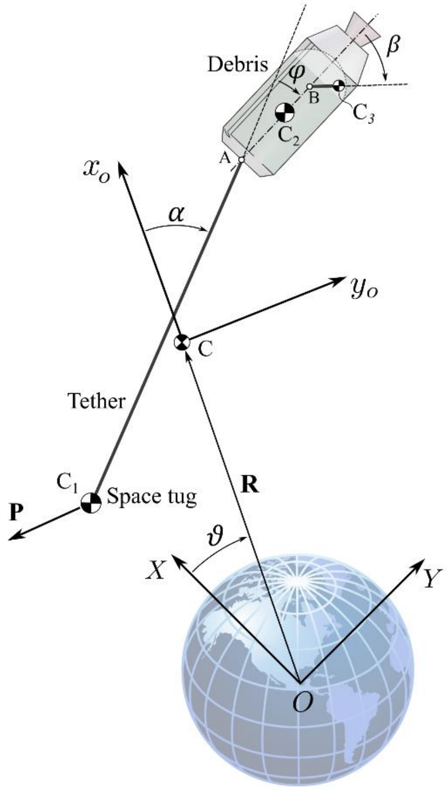

Fig. 1. Tug-debris system

The motion of the system is considered relative to the local vertical local horizontal (LVLH) orbital frame Cx_oy_o. Frame's origin C is in the center of mass of the system. Axis Cx_o lies on the orbital plane and is aligned with the local vertical axis for the center of mass of the system. Axis Cy_o also lies on the orbital plane and directed to the orbital velocity vector of the center of mass.

The motion of the center of mass of the system in Earth centered inertial frame OXY described by two equations

$$\ddot{R} = -\frac{\mu}{R^2} + R\dot{\vartheta}^2 \quad (1)$$

$$\ddot{\vartheta} = -\frac{1}{R}\left[\frac{P}{m} + 2\dot{R}\dot{\vartheta}\right] \quad (2)$$

where $m = m_1 + m_2 + m_3$ is the total mass of the system, P is the thrust of the tug. Vector of the tug's thrust is directed into the opposite direction of the Cy_0 axis.

The Lagrange formalism is used to write the motion equations of the system in non-inertial frame Cx_0y_0

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i,\ i = 1, \dots, 3 \quad (3)$$

where T is the kinetic energy of the system, $q_1 = \alpha$, $q_2 = \varphi$, $q_3 = \beta$ are the generalized coordinates of the system, Q_i is the generalized force for q_i.

The positions of the debris $\boldsymbol{\rho}_2 = [x_2, y_2]^T$, its fuel mass $\boldsymbol{\rho}_3 = [x_3, y_3]^T$ and the tug $\boldsymbol{\rho}_1 = [x_1, y_1]^T$ are described by the following expressions relative to the origin C of the Cx_0y_0 frame

$$\boldsymbol{\rho}_1 = \boldsymbol{\rho}_A - \boldsymbol{A}_\alpha \boldsymbol{e}_x\, l \quad (4)$$

$$\boldsymbol{\rho}_3 = \boldsymbol{\rho}_A + \boldsymbol{A}_\alpha \boldsymbol{A}_\varphi (\boldsymbol{e}_x\, b + \boldsymbol{A}_\beta \boldsymbol{e}_x\, c) \quad (5)$$

$$\boldsymbol{\rho}_2 = \boldsymbol{\rho}_A + \boldsymbol{A}_\alpha \boldsymbol{A}_\varphi \boldsymbol{e}_x\, a \quad (6)$$

where $l = C_1A$ is the tether length, $\boldsymbol{e}_x = [1\ \ 0]^T$, $\boldsymbol{A}_\alpha$, $\boldsymbol{A}_\varphi$, $\boldsymbol{A}_\beta$ are rotation matrixes in OXY plane

$$\boldsymbol{A}_x = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \quad (7)$$

Point C is the center of mass of the system, so we can write

$$\sum_{i=1}^{3} \boldsymbol{\rho}_i m_i = \boldsymbol{0} \quad (8)$$

This equation allows us to express $\boldsymbol{\rho}_A$ for the expressions (1)-(3)

$$\boldsymbol{\rho}_A = \frac{\boldsymbol{A}_\alpha \boldsymbol{A}_\varphi \boldsymbol{e}_x\, a\, m_1 + \boldsymbol{A}_\alpha \boldsymbol{A}_\varphi (\boldsymbol{e}_x\, b + \boldsymbol{A}_\beta \boldsymbol{e}_x\, c)}{m} \quad (9)$$

The kinetic energy of the relative motion of the tug debris and the fuel is given by the expression

$$2T = \sum_{i=1}^{3} m_i \boldsymbol{V}_i^2 + J_z \omega_2^2 \quad (10)$$

where $\boldsymbol{V}_1 = \left[V_{1x}, V_{1y}\right]^T = d\boldsymbol{\rho}_1/dt$, $\boldsymbol{V}_2 = \left[V_{2x}, V_{2y}\right]^T = d\boldsymbol{\rho}_2/dt$, $\boldsymbol{V}_3 = \left[V_{3x}, V_{3y}\right]^T = d\boldsymbol{\rho}_3/dt$.

Generalized force Q_i on coordinate q_i ($i = 1,2,3$) can be written as

$$Q_i = \boldsymbol{P} \cdot \frac{\partial \boldsymbol{\rho}_1}{\partial q_i} + \sum_{k=1}^{3} \boldsymbol{G}_k \cdot \frac{\partial \boldsymbol{\rho}_k}{\partial q_i} + M_i \quad (11)$$

where

$$\boldsymbol{P} = -[0\ \ 1]^T P \quad (12)$$

and M_i is the gravitational torque

$$M_2 \approx -\frac{3\mu}{R^3}(J_z - J_x)\sin 2(\alpha + \varphi) \quad (13)$$

It is supposed in the expression (13) that the tether length is much smaller than the OC distance, so R_2 could be replaced by R.

Only the debris object is considered as a rigid body than only the debris object can be affected by the gravitational torque, so $M_1 = M_3 = 0$. $\boldsymbol{G}_k$ ($k = 1,2,3$) is the force column vector that includes the gravitational force acting on the body k and fiction forces that should be appeared in the system equations due to the non-inertial nature of the orbital frame $Cx_o y_o$ [10]

$$\mathbf{G}_k = m_k \begin{bmatrix} 2\,\mu \dfrac{x_k}{R^3} + 2\dot{\vartheta} V_{ky} + \ddot{\vartheta} y_k + \dot{\vartheta}^2 x_k \\ -\mu \dfrac{y_k}{R^3} - 2\dot{\vartheta} V_{kx} - \ddot{\vartheta} x_k + \dot{\vartheta}^2 y_k \end{bmatrix} \quad (14)$$

for $k = 1,2,3$. The expressions (14) obtained with the assumptions that the tether length is much smaller than the distance from the center of the Earth to the center of mass of the system $l \ll R$ [10].

Using the expressions (4)-(14) one can builds the differential equations (3) of the considered system. Obtained equations are very cumbersome, so these equations are not presented here.

We suppose that chaotic motion of the system can be induced by the oscillations of the space debris object or by the oscillations of the fuel residuals in the tank of the debris object. The debris object is a rigid body that oscillates relative to the tether attachment point “A”. The fuel residuals are represented as the pendulum that oscillates relative to the point “B” of the debris. Both oscillatory motions can induce the chaotic motion of the system. To clearly illustrate the chaotic behavior of the system let us consider simplified model of the system without fuel residuals. This model allows us to write here the motion equations and show the influence of the oscillation of the debris to the attitude motion of the tethered system.

2.2 Motion of the system without fuel residuals

Suppose that the acceleration due to the low thrust tug is small

$$a_\tau = \frac{P}{m} \ll g = \frac{\mu}{R^2} \quad (15)$$

Based on this simplification, the attitude motion of the system can be studied assuming that the geometry of the orbit is preserved. The chaotic motion of the considered system induced by the orbital eccentricity is presented in [11]. Here we focus on the influence of the debris object to the motion of the tethered system, so we suppose that the orbit is circular $R = \text{const}$.

In the expressions (3)-(14) we set $\beta = \dot{\beta} = \ddot{\beta} = 0$, $c = 0$ and write non-dimensional form of the equations using true anomaly angle as the independent variable

$$\begin{aligned} &(1 + 2\,\tilde{a}\cos\varphi + \tilde{J}_z)\alpha'' + (\tilde{J}_z + \tilde{a}\cos\varphi)\varphi'' = \\ &\cos\alpha\left[\frac{P}{m_1 l\, n^2} - 3\sin\alpha\right] + \\ &\tilde{a}\cos(\alpha+\varphi)\frac{P}{m_1 l\, n^2} - 3\tilde{a}\sin(2\alpha+\varphi) + \\ &\tilde{a}\,\varphi'\big(2 + (2\alpha' + \varphi')\big)\sin\varphi - \\ &3\tilde{a}^2\sin(\alpha+\varphi)\cos(\alpha+\varphi) \end{aligned} \quad (16)$$

$$\begin{aligned} &(\tilde{J}_z + \tilde{a}\cos\varphi)\,\alpha'' + \tilde{J}_z\,\varphi'' = \\ &= +\frac{P\tilde{a}\cos(\alpha+\varphi)}{n^2\, m_1 l} \\ &-3[\tilde{a}\cos\alpha + \tilde{J}_{zx}\cos(\alpha+\varphi)]\sin(\alpha+\varphi) \\ &- 2\,\tilde{a}\,\alpha'\sin\varphi \end{aligned} \quad (17)$$

where

$$\tilde{J}_z = \frac{J_z + m_{12}a^2}{m_{12}l^2},\ \tilde{J}_{zx} = \frac{J_z - J_x + m_{12}a^2}{\mathrm{m}_{12}l^2} \quad (18)$$

$m_{12} = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the tug-debris system, $\tilde{a} = a/l$, $\alpha' = \partial\alpha/\partial\vartheta$, $\alpha'' = \partial^2\alpha/\partial\vartheta^2$, $\varphi' = \partial\varphi/\partial\vartheta$, $\varphi'' = \partial^2\varphi/\partial\vartheta^2$, n is the mean motion of the system, that for circular orbit with radius R can be written as

$$n = \sqrt{\frac{\mu}{R^3}} \quad (19)$$

For $\tilde{a} = \tilde{J}_z = \tilde{J}_{zx} = 0$ the equation (16) describes the motion of two masses connected by a tether under the action of the force P

$$\alpha'' = \frac{P}{m_1 l\, n^2} - \frac{3}{2}\sin 2\alpha \quad (20)$$

This equations was presented in [11].

3. Simulation results

In this section we illustrate that the considered system experiences chaotic motion induced by the motion of the debris object relative to the tether.

3.1 Parameters of the system

Parameters of the system are presented in Table 1. The tether of length 500 m connects two bodies with mass 4000 kg (debris) and 1000 kg (space tug). The debris equipped with low thrust propulsion system with thrust of 0.4 N. The system orbiting circular orbit with height of 800 km.

Table 1. Parameters of the system

Parameter	Value
Debris mass m_2, kg	4000
Debris moment of inertia, J_z kg·m^2	10000
Debris moment of inertia, J_x kg·m^2	5000
Space tug mass m_1, kg	1000
Fuel mass m_3, kg	500
$AC_2=a$, m	4
$AB=b$, m	5
$BC_f=c$, m	2
Tether length l, m	500
Tug’s thrust P, N	0.4
Semimajor axis, km	7171
Eccentricity, e	0
Radius of the Earth. R_e, km	6371

3.2 Stationary points

Considered tug-tether-debris system can be represented as a two mass system connected with a massless rod. In central gravitational field it has two stationary points $\alpha = 0$ (stable) and $\alpha = \pi/2$ (unstable). Tug's thrust P shifts stable stationary. This stationary point depends on the tug's thrust, length of the tether and masses of the tug and the debris. Due to small distance from the tether attachment point A to the center of mass of the debris the influence of the debris object as a rigid body to the stationary position of the tether is negligible.

To determine the stationary solutions of the equations (16)-(17), the derivatives are set to zero. That leads to the following equations

$$[\cos\alpha + \tilde{a}\cos(\alpha+\varphi)] \times \left[\frac{P}{m_1 l\, n^2} - 3(\sin\alpha - \tilde{a}\sin(\alpha+\varphi))\right] = 0 \tag{21}$$

$$\frac{P\tilde{a}\cos(\alpha+\varphi)}{n^2\, m_1 l} - 3[\tilde{a}\cos\alpha + \tilde{J}_{zx}\cos(\alpha+\varphi)]\sin(\alpha+\varphi) = 0 \tag{22}$$

for $\tilde{a} = \tilde{J}_{zx} = 0$ we get stationary solutions for two mass system considered in [11]

$$\left[\frac{P}{m_1 l\, n^2} - 3\sin\alpha\right]\cos\alpha = 0 \tag{23}$$

Fig. 2 and 3 show bifurcation diagrams for the angles α and φ as a function of tether length.

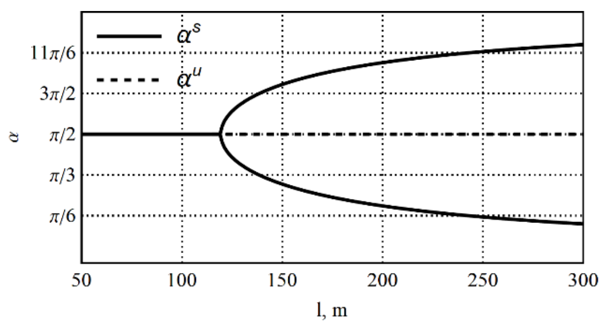

Fig. 2. Stationary solution for α angle as a function of the tether length

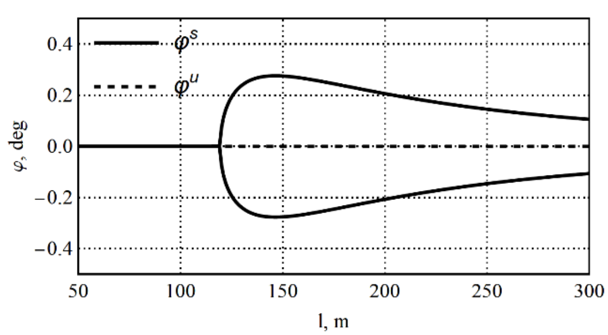

Fig. 3. Stationary solution for φ angle as a function of the tether length

For the full system that includes fuel residuals we should solve nonlinear equations (3) with all the derivatives are set to zero. Solutions of these equations for the angles φ and β are presented in Fig. 4.

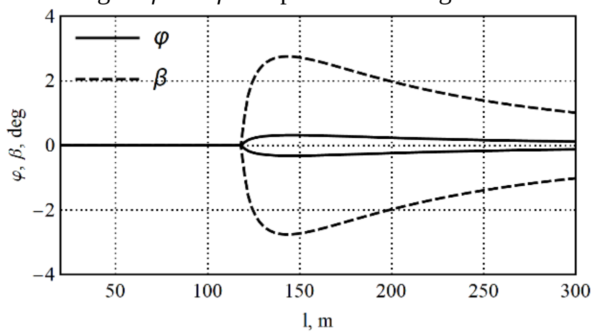

Fig. 4. Stationary solution for φ and β angles as a function of the tether length

3.3 Chaotic motion

In this section we illustrate the chaotic motion of the system that can induced by the oscillations of the debris body relative to the tether attachment point "A".

Fig. 5 shows the evolution of the angle α. The graph illustrates that the motion of the system can be disturbed by the attitude motion of the debris object. The attitude motion of the debris object can induce the chaotic motion of the system near the separatrix. The system starts to oscillate around the stationary point $\alpha^s < \pi/2$ and during the motion transits to oscillation around the stationary point $\alpha^s > \pi/2$ and vice versa.

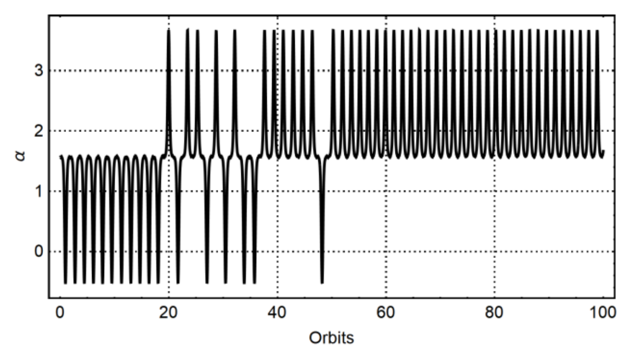

Fig. 5. Chaotic motion of the system without fuel residuals ($\alpha_0 = \pi/2 - 0.02,\ \varphi_0 = 0.5,\ \alpha'_0 = \varphi'_0 = 0$)

The system with fuel residuals undergoes the motion of a like nature. Fig. 6 shows motion of the system with the following initial conditions $\alpha_0 = \pi/2 - 0.02,\ \varphi_0 = \varphi'_0 = \varphi'_0 = 0, \beta_0 = 1$. The oscillation of the fuel leads to the oscillations of the debris body relative to the tether with amplitude about 0.15 rad. Fig. 7 illustrates the motion of the debris body relative to the tether in (φ, φ') phase space during the first two orbits. The oscillations of the angle φ also induce chaotic motion of the system near the unstable point.

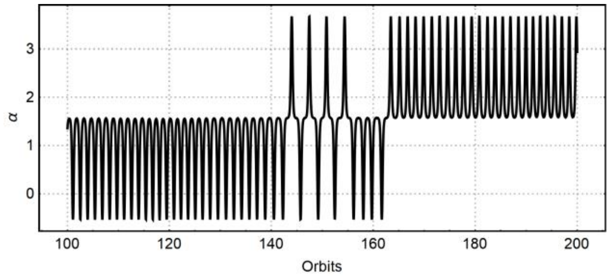

Fig. 6. Chaotic motion of the system with fuel residuals

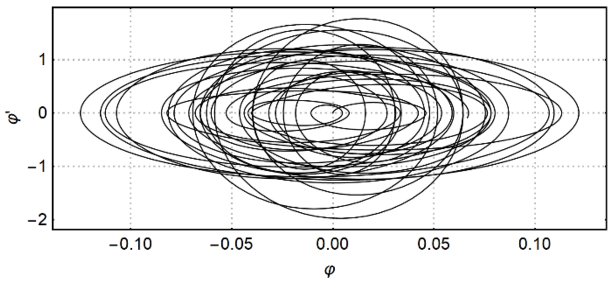

Fig. 7. Motion of the debris relative to the tether

Fig. 8 shows Poincaré maps for four trajectories with the following initial conditions

a. $\alpha_0 = \pi/2 - 0.025$, $\varphi_0 = 1$
b. $\alpha_0 = \pi/2 + 0.002$, $\varphi_0 = 1$
c. $\alpha_0 = \pi/6$, $\varphi_0 = 1$
d. $\alpha_0 = 5\pi/6$, $\varphi_0 = 1$

We can see area of chaotic motion depicted by a diffused set of points near the separatrix (for the first and the second set of initial conditions), which divide the phase space into two oscillation and two rotations areas.

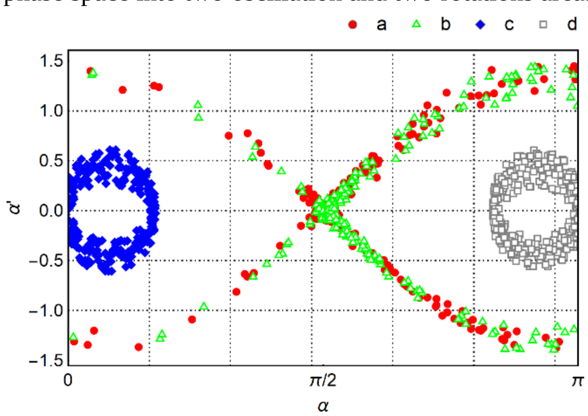

Fig. 8. Poincaré map for P=0.1 N and e=0.05

Fig. 9 plots the largest Lyapunov exponents for three cases. In the first case $\alpha_0 = \pi/2 - 0.025$, $\dot{\alpha}_0 = 0$, $\varphi_0 = 1$, φ'=0 and for $P = 0.4\,N$. In this case the largest Lyapunov exponent tends to a positive value, about 0.2 which indicates the chaotic behavior of the system.

In the second case $\tilde{a} = 0$ $(a = 0)$. The center of mass of the debris body coincides with the tether attachment point "A". In this case the attitude motion of the debris body does not affect the motion of the tether and can't induce chaotic motion of the tethered system. There is no disturbance in the system and the largest Lyapunov exponent tends to zero.

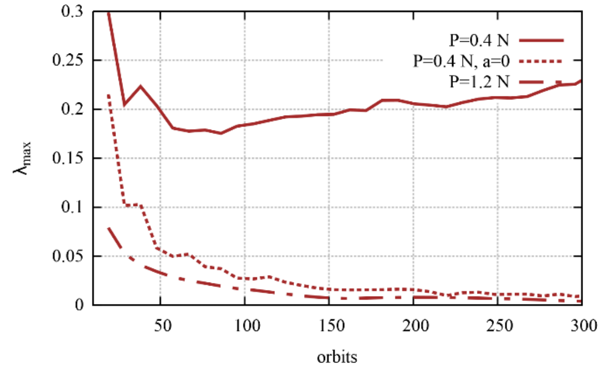

Fig. 9. Largest Lyapunov exponents

The largest Lyapunov exponent also tends to zero if $P = 1.2$ N and $\tilde{a} \neq 0$. In this case the motion starts near the stable point $\alpha^s = \pi/2$, so the motion of the system in this case is not chaotic.

4. Conclusion

The simulation results show that tethered towing using low thrust space tug can lead to chaotic motion of the system if there is an unstable equilibrium of the undisturbed system. The chaotic motion of the system can be induced by the oscillations of the space debris object relative to it's the tether attachment point and by oscillations of fuel residuals.

Acknowledgements

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