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## Dynamic analysis of space tether system with sliding bead-capsule for payload delivery

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### Abstract

Delivery of payload from orbit is an important practical issue. Currently, capsules, which are transferred into the descent orbit by jet engines, are used to solve this problem. An alternative method is based on the applying space tethers. In this paper, space transportation system for the delivery of payloads to the Earth, which is consist of a radially oriented space tether system and a bead-capsule with payload, is considered. The space tether system includes a heavy satellite and a light lower module. The satellite's center of mass moves in a circular orbit. The bead-capsule has cylindrical hole and slides-down along the tether under the influence of the gravitational force and the reaction force of the tether. The capsule separates from the tether near the lower module and passes into the descent orbit. The aim of this work is development of a mathematical model of the described transportation system, analysis of the impact of the bead-capsule motion on the behavior of the space tether system, and choice the system parameters to ensure safe descent.

The plane motion of the system was considered. Equations of motion were constructed using the Lagrange formalism. In the framework of the mathematical model, the satellite was considered as a material point, and the bead-capsule and the lower module were rigid bodies. The tether was modeled by a massless three-segment rod, which takes into account its bending caused by the capsule motion. The influence of the initial velocity of the capsule, its mass, and a coefficient of friction between the capsule and the tether on the system's oscillations were investigated. It was shown that, the segment of the tether located below the capsule can swing-up and even turn into rotation as a result of the capsule uncontrolled sliding. Parameters of the system were chosen to prevent the occurrence of this undesirable situation. It was shown that high-frequency oscillations of the capsule around its center of mass can occur during its sliding. The effects of the moment of the capsule separation from the tether, its mass and initial velocity on the height of the perigee of the capsule orbit were analyzed. The results of the work can be used to design new space transport systems based on long tethers and moving capsules.

**Keywords:** space tethered system, climber, space elevator, bead-capsule, deorbiting

### 1. Introduction

Space tethered systems is a promising direction of modern astronautics. This technology will allow to reduce the cost of many transport operations in space by eliminating the reactive engines use. A review of the possible space tethers applications can be found in [1-4]. In this study, we focus on the issue of the payload delivery from orbit to the Earth surface. In the scientific literature several tether-based approaches are considered to solve this problem. The orbital velocity of the payload re-entry capsule can be reduced to transfer it to the descent trajectory by redistributing the angular momentum between the tethered bodies in the case of their rotation [5, 6]. Descent from an orbital station or satellite can be carried out by a variable length tether using various control laws [7-9]. An electrodynamic tether attached to the capsule with a payload can be used

for braking its motion [10, 11]. The payload can be transported by means of an auxiliary spacecraft-tug attached to it by a tether [12, 13]. The space elevator is one of the most complex and ambitious projects of connecting the Earth's surface with the orbit by a tether-based transport channel [14-16]. The reusable payload delivery system considered in this paper is a development of the space elevator idea.

Considered space transportation system consists of a radially oriented space tether system, which exists in orbit for a long time, and a bead-capsule with payload, that can slide on the tether. The space tether system includes a heavy satellite and a light lower module. The satellite moves in a circular orbit. The bead-capsule has cylindrical hole and slides-down along the tether under the influence of the gravitational force and the reaction force of the tether. The payload deorbiting operation

can be divided into several stages. The capsule is separated from the satellite by spring pushers and moves for a while along the rigid guides (Fig 1a, Fig 2). After the descent from the guides, the capsule slides along the tether. The Coriolis inertial force, acting on the capsule in the noninertial rotating orbital reference frame of the satellite, deflects it in the direction of the system orbital flight. The reaction force from the tether does not allow the capsule to deviate significantly from the local vertical (Fig 1b). It provides decrease in the capsule orbital velocity. A few meters before the lower module, the capsule separates from the tether (Fig 1c). For example, this can be realized by dividing the capsule into several independent parts (Fig 3). After separation the orbital velocity of the capsule is insufficient to keep it in orbit, and the capsule enters the atmosphere (Fig 1d). The space tether system can be used to lower the next bead-capsule after the tether oscillations damping and the system transferring into its initial stable radial state.

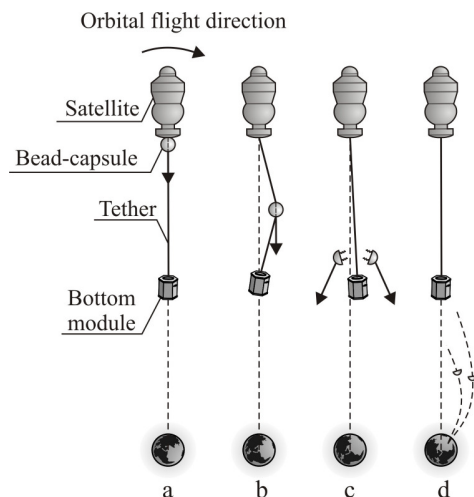


Fig. 1. The payload deorbiting operation stages

The purpose of this work is development of a mathematical model of the described transportation system, analysis of the impact of the bead-capsule motion on the behavior of the space tether system, and choice the system parameters to ensure safe descent. This study continues the researches published in [17, 18]. In contrast to existing works dealing with the motion of a material point along the tether [15, 19, 20], in this study the capsule is considered as a rigid body. Its motion is given dynamically taking into account the friction force.

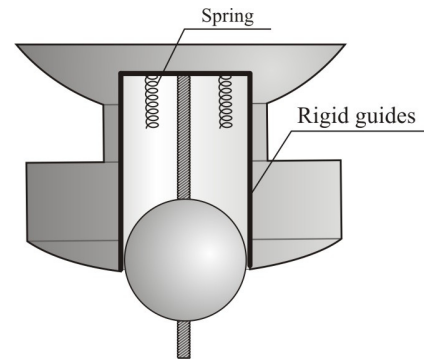


Fig. 2. The bead-capsule motion along the guides

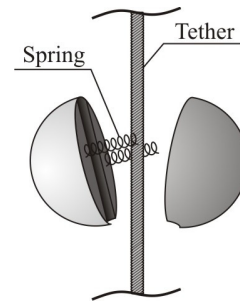


Fig. 3. Dividing the capsule

The paper consists of four sections. The state of art and paper purpose are given in Introduction. The second section is devoted to the mathematical model development. The results of numerical simulation and their discussion are given in the third section. The fourth section contains conclusions.

## 2. Mathematical model

Considered mechanical system consists of a satellite, a bead-capsule, a bottom module and a tether (Fig. 4). It is supposed that the satellite is a material point which moves on a circular orbit. The bead-capsule and the module are rigid bodies. The tether is a massless inelastic three-segment rod. It is assumed that motion is plane. The mechanical system can be described by six generalized coordinates

$$\mathbf{q} = (\nu, \varphi_1, \varphi_2, \varphi_3, \varphi_4, s_1), \quad (1)$$

where  $\nu$  is the true anomaly angle of the satellite,  $\varphi_1$ ,  $\varphi_2$  and  $\varphi_3$  are the angles of the tether segments deflection from the satellite local vertical,  $s_1$  is the length of the tether segment between the satellite and the bead-capsule.

Equations of motion of the considered system can be constructed with the help of the Lagrange equations of the second kind

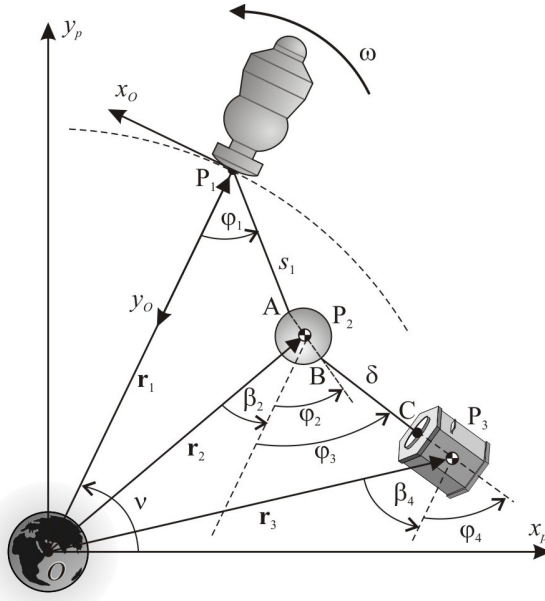


Fig. 4. Mechanical system and generalized coordinates

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j, \quad (2)$$

where  $L$  is Lagrange function,  $Q_j$  is the generalized nonpotential forces. Lagrange function of the considered mechanical system has the form

$$L = \sum_{i=1}^3 \frac{m_i (\dot{x}_i^2 + \dot{y}_i^2)}{2} + \frac{J_{z2} (\dot{\varphi}_2 + \omega)^2}{2} + \frac{J_{z3} (\dot{\varphi}_4 + \omega)^2}{2} + \mu \sum_{i=1}^3 \frac{m_i}{r_i} + \frac{\mu}{2} \sum_{i=2}^3 \frac{J_{xi} + J_{yi} + J_{zi}}{r_i^3} + \frac{3\mu J_{x2} \cos^2(\varphi_2 + \beta_2) + J_{y2} \sin^2(\varphi_2 + \beta_2) + J_{z2}}{2r_2^3} + \frac{3\mu J_{x3} \cos^2(\varphi_4 + \beta_3) + J_{y3} \sin^2(\varphi_4 + \beta_3) + J_{z3}}{2r_3^3}, \quad (3)$$

where index  $i=1$  corresponds to the satellite,  $i=2$  denotes the bead-capsule, and  $i=3$  marks the bottom module,  $m_i$  is the mass of the  $i$ -th body,  $\mathbf{r}_i = [x_i, y_i]$  is the radius vector of the center of mass  $P_i$  of the  $i$ -th body (Fig. 4),  $J_{xi}$ ,  $J_{yi}$ ,  $J_{zi}$  are the moments of inertia of the  $i$ -th body,  $\omega$  is the angular velocity of the satellite,  $\beta_i$  is the angle between the vectors  $\mathbf{r}_1$  and  $\mathbf{r}_i$ ,  $\mu$  is the gravitational constant of the Earth. The coordinates of the  $i$ -th body center of mass can be given in a inertial coordinate system  $Ox_p y_p$

$$x_1 = r_1 \cos v, \quad y_1 = r_1 \sin v, \quad (4)$$

$$x_2 = x_1 - \sum_{i=1}^2 s_i \cos(\varphi_i + v), \quad y_2 = y_1 - \sum_{i=1}^2 s_i \sin(\varphi_i + v), \quad (5)$$

$$x_3 = x_2 - \sum_{i=2}^4 s_i \cos(\varphi_i + v), \quad y_3 = y_2 - \sum_{i=2}^4 s_i \sin(\varphi_i + v), \quad (6)$$

where  $s_2 = AP_2 = P_2B$  is the bead-capsule radius,  $s_3 = BC$  is the lower tether segment length,  $s_4 = CP_3$  is the distance between the tether attachment point and the bottom module center of mass,  $v = \omega t$ .

The generalized forces  $Q_j$ , caused by the friction between the tether and the bead-capsule, can be found using their definition

$$Q_j = \mathbf{F}_{fr} \cdot \frac{\partial \mathbf{r}_2}{\partial q_j}, \quad (7)$$

where  $F_{fr} = \mu_f N \text{sign}(\dot{s}_1)$  is the friction force,  $\mu_f$  is the friction coefficient,  $N$  is the normal force. Taking into account that the friction force is directed along the line AB (Fig.4) and using equations (5), the general forces can be written as

$$Q_{\varphi_1} = Q_{\varphi_2} = Q_{\varphi_3} = Q_{\varphi_4} = 0, \quad Q_{s_1} = -\mu_f N \text{sign}(\dot{s}_1). \quad (8)$$

The unknown normal force can be found from the Newton's second law for the capsule.

$$m_2 \ddot{\mathbf{r}}_2 = \mathbf{G} + \mathbf{F}_{fr} + \mathbf{N}, \quad (9)$$

where  $G = \mu m_2 r_2^{-2}$  is the gravitational force (Fig. 5). Projecting the vector equation (9) on the axis of the inertial reference frame  $Ox_p y_p$  and following expression of the normal force  $N$  from it considering the formulas (5) gives

$$N = m_2 \left( \ddot{\varphi}_1 s_1 \cos(\varphi_1 - \varphi_2) + \ddot{\varphi}_2 s_2 + \ddot{s}_1 \sin(\varphi_1 - \varphi_2) - s_1 (\omega + \dot{\varphi}_1)^2 \sin(\varphi_1 - \varphi_2) + 2\dot{s}_1 (\omega + \dot{\varphi}_1) \cos(\varphi_1 - \varphi_2) - r_1 \omega^2 \sin \varphi_2 + \frac{\mu}{r_2^3} (r_1 \sin \varphi_2 - s_1 \sin(\varphi_1 - \varphi_2)) \right). \quad (10)$$

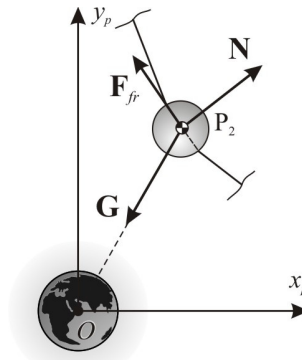


Fig. 5. Forces acting on the bead-capsule

Substitution of the expressions (3), (8), and (10) in (2), gives a system of five second-order differential equations describing the motion of a space tethered system with a sliding bead-capsule.

### 3. Results of numerical simulations

Consider the payload descent from the orbit by means of a transport system having the following parameters: the mass of the satellite is  $m_1 = 5000 \text{ kg}$ , the mass of the bead-capsule is  $m_2 = 50 \text{ kg}$ , the mass of the bottom module is  $m_3 = 100 \text{ kg}$ , the tether length is  $l = 30 \text{ km}$ , the friction coefficient is  $\mu_f = 0.1$ , the height of the satellite's orbit 300 km, the radius of the capsule is  $s_2 = 0.25 \text{ m}$ , the distance between the bottom module center of mass and tether attachment point is  $s_4 = 1 \text{ m}$ , the moments of inertia of the bead-capsule are  $J_{x2} = J_{y2} = J_{z2} = 1.25 \text{ kg} \cdot \text{m}^2$ , the bottom module moments of inertia are  $J_{x3} = 12 \text{ kg} \cdot \text{m}^2$ ,  $J_{y3} = J_{z3} = 35 \text{ kg} \cdot \text{m}^2$ . The spring pushers give the capsule the initial velocity  $\dot{s}_{10} = 3 \text{ m/s}$ . The length of guide channel is  $s_{10} = 1 \text{ m}$  (Fig. 2).

Fig. 6 shows dependences of the system's deflection angles  $\varphi_i$  on time. As can be seen from the figure, under the influence of the Coriolis force, the tether deviates from the satellite's local vertical in the direction of the orbital flight ( $\varphi_1$ ). This effect is known from the previous studies [20,21]. The Coriolis force also leads to the occurrence of the high-frequency oscillations of the bead-capsule relative its center of mass ( $\varphi_2$  on Fig. 7). The amplitude of these oscillations is the greater, the smaller the mass of the capsule. To reduce this amplitude, a guide channel can be used (Fig. 2). It prevents the tether and capsule deviating from the local vertical some time after the bead-capsule separation from the satellite. Fig 8 demonstrates the effect of the channel length  $s_{10}$  on the capsule oscillation. The length of this channel determines the maximum allowable angle of the tether deflection from the local vertical  $\varphi_1$ .

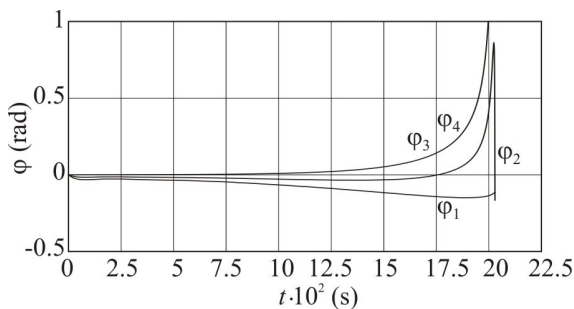


Fig. 6. Dependences of the deflection angles  $\varphi_i$  on time

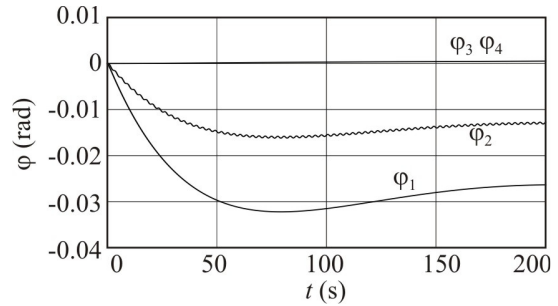


Fig. 7. Dependences of the deflection angles  $\varphi_i$  on time

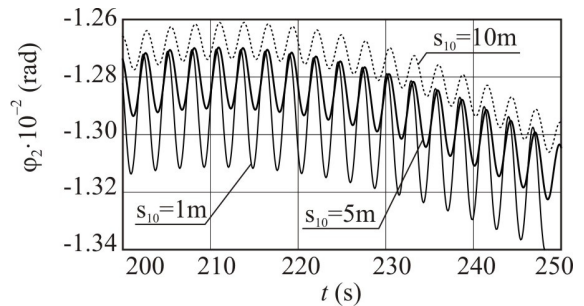


Fig. 8. Dependence of the bead-capsule oscillation on time for various guide channel lengths

It is necessary to note the effect of the transition of the tether below the capsule into rotation as the capsule approaches the lower end of the tether. A sharp increase in the amplitude of oscillations of the lower tether segment was described in the works [20, 21], but in contrast to these studies the effect is more pronounced because the capsule relative velocity  $\dot{s}_1$  increases, rather than constant (Fig. 9). An increase in the angle of deflection of the tether lower segment leads to a sharp increase in normal force  $N$ . As a result the capsule slows down. In addition, increasing the angle  $\varphi_3$  leads to a jamming when the bottom module occurs above the capsule, and to continue the movement, the capsule must climb upward.

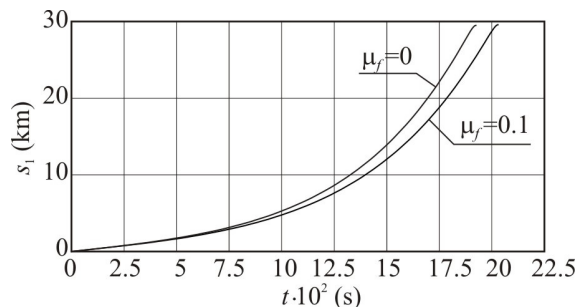


Fig. 9. Dependence of the bead-capsule position on time

The friction force has no fundamental effect on the character of the system motion. Increasing the coefficient of friction  $\mu_f$  leads to a decrease in the

bead-capsule relative velocity (Fig. 9). This in turn leads to a decrease in the Coriolis force and consequently to a decrease in the angles of deviation from the local vertical. The entanglement of the tether occurs later than in the case without friction.

The bead-capsule must be separated from the tether before the before its entanglement. The Fig. 10, 11 show the perigee radius of the capsule orbit for different masses of the capsule and its initial velocities. The angle 45 deg was taken as maximum allowable deflection angle for  $\varphi_3$ . When this angle is reached, the capsule should be automatically separated to prevent the tether entanglement and subsequent accidents. The radius of the perigee of the capsule's orbit  $r_\pi$  can be considered as a efficiency criterion of the descent maneuver. Formulas for its calculation are given in [22]. Calculations showed that for the descent scheme under consideration, the later the capsule separates from the tether, the smaller the radius of the perigee of the capsule's orbit. Fig. 11 demonstrates that increasing the initial speed of the capsule does not always lead to a decrease in the radius of the perigee of its orbit.

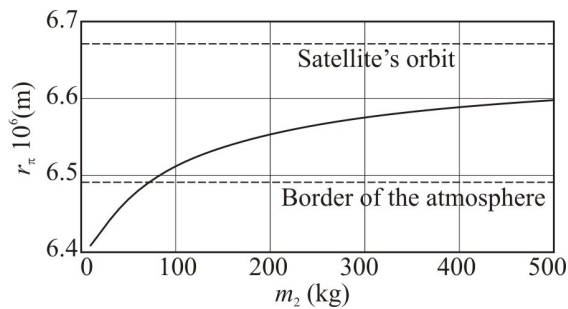


Fig. 10. The influence of the capsule mass on the radius of the perigee of its orbit

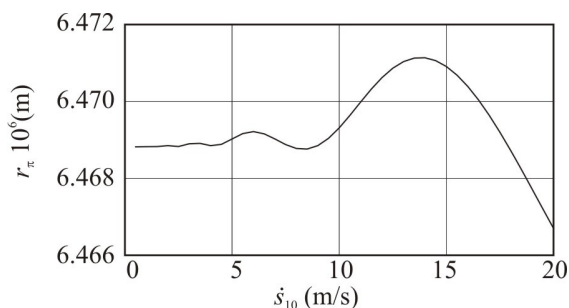


Fig. 11. The influence of the capsule initial relative velocity on the radius of the perigee of its orbit

Fig 10 shows that an increase in the mass of the bead-capsule leads to a decrease in its perigee radius. As the mass increases, the Coriolis force begins to exert a greater influence on the system's motion. This leads to the fact that the tether's segments deviate to large angles Fig 12, and the tether entangling occurs earlier. In this

regard it is necessary to separate the capsule from the tether at a greater distance from the lower module. An increase in the altitude at which the separation from the tether occurs is the main factor determining the perigee radius increase when the mass of the capsule increases.

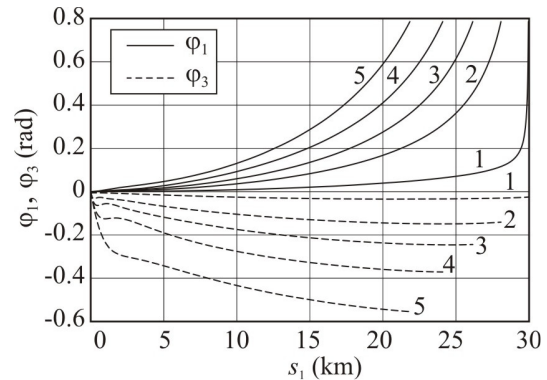


Fig. 12. Dependences of the tether's segments deflection angles on the capsule position for various masses  $m_2$ : case 1 – 10kg, case 2 - 50kg, case 3 – 100 kg, case 4 – 200kg, case 5-500 kg

Results of numerical simulations given in this section confirm the possibility of the considered transport system using to solve the problem of payload delivery from orbit to the Earth. To ensure the safe descent of the capsule from the orbit, it is necessary to provide its separation before the lower segment of the tether goes into rotation. The mass of the capsule has a strong influence on the perigee radius of its orbit after separation. Therefore, for successful descent, the mass of the capsule should be chosen small enough to make the perigee radius less than the atmosphere boundary radius.

#### 4. Conclusions

In this paper the dynamics of the reusable payload delivery system was studied. The mathematical model was constructed using Lagrange formalism. A distinctive feature of the developed model is the consideration of the descent capsule's motion relative its center of mass during sliding along the tether, as well as the allowance the frictional force between the tether and the capsule. It was shown that, the tether segment below the capsule can swing-up and even turn into rotation as a result of the capsule sliding. Another observed effect is the presence of high-frequency oscillations of the capsule relative its center of mass when it descends. The effects of the moment of the capsule separation from the tether, its mass and initial velocity on the height of the perigee of the capsule orbit were analysed. It was shown that the proposed transport system is most effective for the descent of light capsules. The results of the work can be used to design

new space transport systems based on long tethers and moving capsules.

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