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Spatial Dynamics and Attitude Control During Contactless Ion Beam Transportation in GEO

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The development of contactless space debris removal systems is a promising area of modern astronautics. The ion beam removal method is considered in this paper. The aims of the work are study of the dynamics of 3D motion of axisymmetric space debris during its contactless transportation in GEO; the choice of a favorable attitude motion mode of the space debris transportation; and developing control law providing stabilization of the space debris in this mode. A mathematical model of the mechanical system 3D motion consisting of the space debris and an active spacecraft is built. It is proposed to transport the space debris in angular motion mode corresponding to the equilibrium position or to the regular precession mode. Contactless transportation in these modes is more preferable than transportation in arbitrary rotation mode from the point of view of minimizing the efforts spent by the control system of the active spacecraft. The control law for the velocity of ions in the ion beam, which provides stabilization of the space debris in regular precession mode, is proposed. Numerical simulation of the motion of axisymmetric space debris close in shape to Meteosat-8 satellite at the GEO is performed.

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I. Introduction

THE space debris removal is one of the most important tasks of modern astronautics [1, 2]. The mutual collisions of space objects can lead to the formation of large clouds of space debris fragments and make the practical use of near space impossible. The scientific literature discusses many different ways of large space debris removal. Most of them are based on the use of a propulsion system of an external active spacecraft - cleaner. There are projects involving the docking or capture of a space debris object by a robotic manipulator [3-5], harpoon [6, 7], net [8-10], and its subsequent towing on a rigid bundle or on an elastic tether [11, 12]. The direction of contactless space debris transportation is promising. It involves the use of electrostatic [13, 14], gravitational [15, 16], magnetic fields [17], as well as the flow of high-speed particles created by the active spacecraft [18, 19] to exert a force on a passive object. The lack of mechanical contact between the active spacecraft and space debris increases the safety of the active space debris removal system and makes it possible to transport rapidly rotating objects.

The space debris removal by ion beam involves the installation of an additional electrodynamic engine on board the active spacecraft. This engine creates a stream of ions that blows the space debris. Ions hit the surface of space debris and generate a force. The resulting force generated by the ion beam on the entire surface will be called the ion beam force. This force is used to transport space debris. The idea of this space debris removal method was expressed independently of each other by C. Bombardelli, J. Pelaez and S.Kitamura in the works [18, 20]. One of the main disadvantages of this technology is the need to compensate for the thrust created by the ion engine. There is a project of an economical double-sided ion engine aimed at solving this problem [21]. Feasibility of using one compensation thruster to control the in-plane relative position of space debris in eccentric orbits was studied in [22].

The complexity of the ion beam transportation method lies in the fact that the magnitude and direction of the resulting ion beam force depends, among other things, on the position and orientation of the space debris object inside the ion beam. The point of application of the resultant force does not coincide with the center of mass of the object. This leads to the appearance of an ion beam torque relative to the center of mass, which tends to rotate the object in the ion flow. This rotation, in turn, will lead to a change in the magnitude and direction of the ion beam force. To date, studies of the dynamics of space debris during its ion transportation, taking into account the influence of the angular orientation on the magnitude of the ionic impact, have been carried out in a planar statement [19, 23, 24]. The planar motion case is ideal and cannot be realized in practice, however, the revealed patterns of motion and found control laws can be useful in the analysis of the general case of 3D motion. A study of the space debris motion as a material point without taking into account its rotation in the ion beam was performed in the works [18, 20, 25]. These studies have shown the feasibility and great practical potential of the ion beam removal method. The work [26] devoted to the development of a methodology of the active spacecraft's parameters synthesis for ion beam removal mission. Speaking about studies of ion beam transportation, one cannot fail to note a monograph [27] that includes interesting scientific results and mathematical models.

The aims of the work are study the dynamics of 3D motion of axisymmetric space debris during its ion beam contactless transportation in a geosynchronous orbit (GEO); the choice of a favorable attitude motion mode of the space debris for its contactless transportation; and developing control law providing stabilization of the space debris in this mode. To achieve these goals, a mathematical model of the mechanical system 3D motion consisting of the space debris and an active spacecraft is built. The calculation procedure described in [19, 27] is used to calculate the ion beam force and torque. For numerical simulation, a solid body close in shape and parameters to the satellite Meteosat-8 [28] is used. This study is a continuation of [24] where a plane motion case is considered.

The paper is consists of six sections. After the introduction, which is the first section, there is a section devoted to the development of a mathematical model. The third section is devoted to the choice of the space debris angular motion mode during contactless transportation. The fourth section proposes the control law of ion engine thrust to stabilize the space debris attitude motion. The fifth section presents the results of numerical simulation of space debris motion with uncontrolled blowing and using the proposed control law. The sixth section contains the main results and conclusions. The authors declare that they have no conflict of interest. The contribution of the authors to this study can be described as follows. Sections 1-4 were written by V. Aslanov and A. Ledkov; Sections 5 and 6 were written by all three authors jointly. The control law given in Section 4 was proposed by A. Ledkov.

II. Mathematical models and methods

A mathematical model of a mechanical system consisting of a space debris object and an active spacecraft in three-dimensional space is developed in this section. It is assumed that the active spacecraft is a material point A, and the space debris object is a rigid body with a center of mass at point B (Fig. 1). The active spacecraft is equipped with control engines and an ion engine, creating an ion beam. The motion of the system occurs only under the influence of the gravitational field of the Earth, the thrust of the active spacecraft engines, and the force and the torque generated by the ion beam when interacting with the surface of the space debris. It is possible, within certain limits, to change the modulus of this force by changing the flow velocity and the distance between space debris and the active spacecraft.

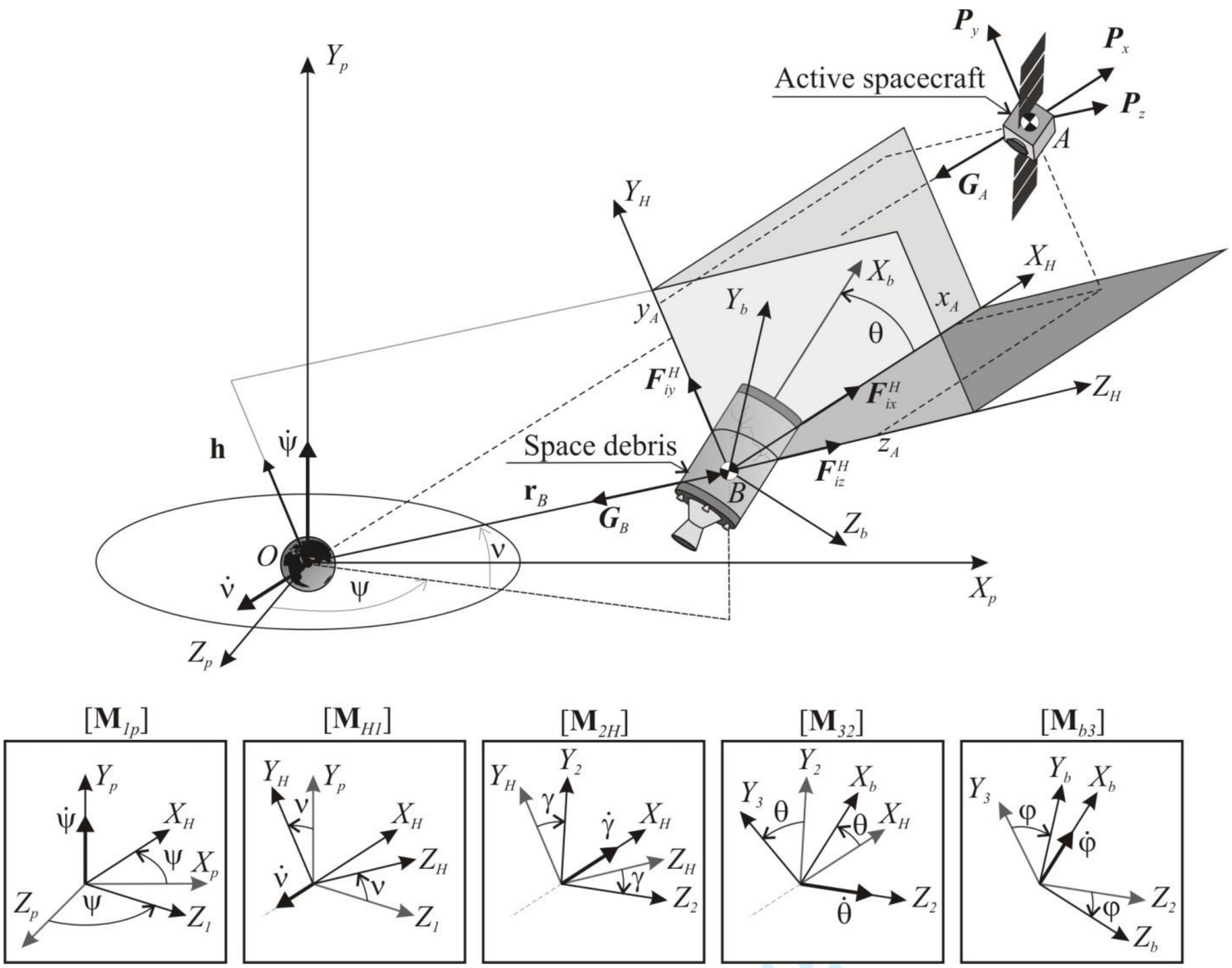

Fig. 1 Reference frames.

A. Coordinate systems and rotation matrices

To obtain the equations of motion several reference frames should be introduced. A planetary frame, whose origin O is in the center of the Earth, is inertial coordinate system. The plane OX_pZ_p is equatorial, the axis OY_p lies on the axis of the Earth rotation. To describe the motion of the active spacecraft relative the space debris object the Hill coordinate frame $BX_HY_HZ_H$ is used. The axis BZ_H is directed through the space debris object's position vector $\mathbf{r}$, the axis BY_H is parallel to the orbit momentum vector $\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}}$, and the axis BX_H complies right-hand system [29]. The body frame $BX_bY_bZ_b$ is fixed relative to the space debris. Its axes are principal body axes. Transformation from inertial frame to Hill frame can be defined by rotation matrices $[\mathbf{M}_{Hp}]$

$$[\mathbf{M}_{Hp}] = \begin{bmatrix} c_\psi & 0 & -s_\psi \\ -s_\psi\, s_\nu & c_\nu & -c_\psi\, s_\nu \\ s_\psi\, c_\nu & s_\nu & c_\psi\, c_\nu \end{bmatrix}, \tag{1}$$

where angles ψ and ν are shown on Fig. 1, $c_\alpha = \cos\alpha$, $s_\alpha = \sin\alpha$. The inverse transition can be made using the matrix $[\mathbf{M}_{pH}] = [\mathbf{M}_{Hp}]^{-1} = [\mathbf{M}_{Hp}]^T$. Transformation from Hill frame to body frame can be defined by rotation matrix $[\mathbf{M}_{bH}]$, which is

$$[\mathbf{M}_{bH}] = \begin{bmatrix} c_\theta & s_\theta c_\gamma & s_\theta s_\gamma \\ -c_\varphi\, s_\theta & c_\varphi c_\theta c_\gamma - s_\varphi s_\gamma & c_\varphi c_\theta s_\gamma + s_\varphi c_\gamma \\ s_\varphi\, s_\theta & -s_\varphi c_\theta c_\gamma - c_\varphi s_\gamma & -s_\varphi c_\theta s_\gamma + c_\varphi c_\gamma \end{bmatrix}, \tag{2}$$

where γ, θ, φ are Euler angles (Fig. 1). Rotation matrix $[\mathbf{M}_{pb}]$ that defines the transition from the body frame $OX_bY_bZ_b$ to the inertial coordinate system $OX_pY_pZ_p$ can be found as

$$[\mathbf{M}_{pb}] = [\mathbf{M}_{pH}][\mathbf{M}_{Hb}] = [\mathbf{M}_{Hp}]^T[\mathbf{M}_{bH}]^T. \tag{3}$$

The superscript of a vector denotes the coordinate system in which the components of the vector are specified. The following notation is used in this paper: “b” is body reference frame, “p” is planetary inertial reference frame, “H” is Hill coordinate frame.

B. Equations of the space debris translational 3D motion

The motion of the center of mass of space debris can be described by Newton's law

$$m_B \ddot{\mathbf{r}}^p = \mathbf{G}_B^p + \mathbf{F}_I^p. \tag{4}$$

where m_B is the mass of the space debris, $\mathbf{r}^p$ is the position vector of the space debris center of mass, $\mathbf{G}_B^p = -\frac{\mu m_B}{r^3}\mathbf{r}^p$ is the gravitational force, μ is gravitational constant of the Earth, $\mathbf{F}_i^H = [F_{ix}^H, F_{iy}^H, F_{iz}^H]^T$ is ion beam force given by its components in Hill frame. The position vector has following components in inertial frame

$$\mathbf{r}^p = [r\sin\psi\cos\nu,\ r\sin\nu,\ r\cos\psi\cos\nu]^T. \tag{5}$$

Calculating the derivatives and solving equation (4) for the second derivatives, we obtain

$$\ddot{r} = r(\dot{\nu}^2 + \dot{\psi}^2 c_\nu{}^2) - \frac{\mu}{r^2} + \frac{F_{iz}^H}{m_B},$$
$$\ddot{\psi} = \frac{2\dot{\psi}\dot{\nu}s_\nu}{c_\nu} - \frac{2\dot{\psi}\dot{r}}{r} + \frac{F_{ix}^H}{rm_B c_\nu}, \quad . \tag{6}$$
$$\ddot{\nu} = -\dot{\psi}^2 s_\nu c_\nu - \frac{2\dot{\nu}\dot{r}}{r} + \frac{F_{iy}^H}{rm_B}.$$

The resulting system of equations describes the motion of the center of mass of space debris. The projections of the ion beam force depend on the relative position of the active spacecraft, the direction of ion beam, and the orientation of the space debris object. It should be noted that position $\nu = \pi n/2$, where $n \in \mathbb{Z}$, is a singular point and equations (6) cannot be used in this case.

C. Equations of the space debris attitude motion

The motion of space debris relative to the center of mass can be described using Euler's equations [29]

$$\frac{d\mathbf{H}_B^b}{dt} + \boldsymbol{\omega}^b \times \mathbf{H}_B^b = \mathbf{L}_G^b + \mathbf{L}_I^b \ , \tag{7}$$

where $\mathbf{H}_B^b = [\mathbf{I}]\boldsymbol{\omega}^b$ is the angular momentum vector about its mass center B. The components of this vector is defined in the body reference frame. $\mathbf{L}_G^b$ is the gravity gradient torque, $\mathbf{L}_I^b$ is the ion beam torque relative to the space debris object's center of mass, $\boldsymbol{\omega}^b$ is the angular velocity of the space debris given by its components in the body reference frame

$$\boldsymbol{\omega}^b = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\varphi} + \dot{\gamma}c_\theta - \dot{\nu}c_\theta + \dot{\psi}(c_\nu s_\theta c_\gamma + s_\nu s_\theta s_\gamma) \\ \dot{\theta}s_\varphi - \dot{\gamma}s_\theta c_\varphi + \dot{\nu}c_\varphi s_\theta + \dot{\psi}(c_\nu(c_\varphi c_\theta c_\gamma - s_\varphi s_\gamma) + s_\nu(s_\varphi c_\gamma + c_\varphi c_\theta s_\gamma)) \\ \dot{\theta}c_\varphi + \dot{\gamma}s_\theta s_\varphi - \dot{\nu}s_\varphi s_\theta + \dot{\psi}(-c_\nu(s_\varphi c_\theta c_\gamma + c_\varphi s_\gamma) + s_\nu(c_\varphi c_\gamma - s_\varphi c_\theta s_\gamma)) \end{bmatrix}. \tag{8}$$

The space debris inertia matrix is the diagonal matrix

$$[\mathbf{I}] = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}, \tag{9}$$

where I_x, I_y, I_z are principle moments of inertia of the space debris object.

The gravity gradient torque acting on the space debris in an inverse square gravity field is defined by the equation [29]

$$\mathbf{L}_G^b = \frac{3\mu}{r^5}\mathbf{r}^b \times [\mathbf{I}]\mathbf{r}^b = \frac{3\mu}{r^3}\begin{bmatrix}(I_z - I_y)\bar{r}_y\bar{r}_z \\ (I_x - I_z)\bar{r}_x\bar{r}_z \\ (I_y - I_x)\bar{r}_x\bar{r}_y\end{bmatrix}, \tag{10}$$

where center of mass vector $\mathbf{r}^b$ is given in the body frame

$$\mathbf{r}^b = [M_{bH}]\begin{bmatrix}0\\0\\r\end{bmatrix} = r\begin{bmatrix}s_\theta s_\gamma \\ c_\varphi c_\theta s_\gamma + s_\varphi c_\gamma \\ -s_\varphi c_\theta s_\gamma + c_\varphi c_\gamma\end{bmatrix} = r\begin{bmatrix}\bar{r}_x\\ \bar{r}_y\\ \bar{r}_z\end{bmatrix}, \tag{11}$$

The ion beam torque is given by its components in body frame $\mathbf{L}_I^b = [L_{Ix}, L_{Iy}, L_{Iz}]^T$. Equation (7) can be reduced to

$$\begin{aligned}\dot{\omega}_x &= \frac{I_y - I_z}{I_x}\left(\omega_y\omega_z - \frac{3\mu}{r^3}\bar{r}_y\bar{r}_z\right) + \frac{L_{Ix}}{I_x},\\ \dot{\omega}_y &= \frac{I_z - I_x}{I_y}\left(\omega_x\omega_z - \frac{3\mu}{r^3}\bar{r}_x\bar{r}_z\right) + \frac{L_{Iy}}{I_y},\\ \dot{\omega}_z &= \frac{I_x - I_y}{I_z}\left(\omega_x\omega_y - \frac{3\mu}{r^3}\bar{r}_x\bar{r}_y\right) + \frac{L_{Iz}}{I_z}.\end{aligned} \tag{12}$$

Equations (8), (12) describe the motion of the space debris object relative to the center of mass under the action of the gravitational and the ion beam torques.

Consider the case of motion of a space debris object with small asymmetry. It is assumed that the dimensionless difference of the moments of inertia Δ is a small quantity

$$\Delta = \frac{I_z - I_y}{I}, \tag{13}$$

where $I = (I_y + I_z)/2$. In this case $I_y = I(1+\Delta/2)$, $I_z = I(1-\Delta/2)$. Following the approach described in the book [30], new variables are introduced based on the classical Lagrange case of motion of a body with a fixed point. In the Lagrange case the generalized momentum corresponding to rotation and precession angles are integrals of motion. In the case of perturbed motion, these quantities will be slowly changing functions

$$R = \bar{I}_x\omega_x,\ G = R\cos\theta + (\omega_z\sin\varphi - \omega_y\cos\varphi)\sin\theta, \tag{14}$$

where $\bar{I}_x = I_x / I$. Expression the angular velocities from (14) and (8) gives

$$\omega_x = \frac{R}{\bar{I}_x},\quad \omega_y = \dot{\theta}s_\varphi - \frac{(G - Rc_\theta)c_\varphi}{s_\theta} - \dot{\psi}s_\varphi(c_\nu s_\gamma - s_\nu c_\gamma),\quad \omega_z = \dot{\theta}c_\varphi + \frac{(G - Rc_\theta)s_\varphi}{s_\theta} - \dot{\psi}c_\varphi(c_\nu s_\gamma - s_\nu c_\gamma), \tag{15}$$

$$\dot{\gamma} = \frac{G - Rc_\theta}{s_\theta^2} + \dot{\nu} + \frac{\dot{\psi}c_\theta(c_\nu c_\gamma + s_\nu s_\gamma)}{s_\theta}, \tag{16}$$

$$\dot{\varphi} = \frac{R}{I_x} - \frac{(G - Rc_\theta)c_\theta}{s_\theta^2} - \frac{\dot{\psi}(c_\nu c_\gamma + s_\nu s_\gamma)}{s_\theta}. \tag{17}$$

After substituting equations (15) in (12), expressing the derivatives we obtain

$$\begin{aligned}\ddot{\theta} + \frac{(G - R\cos\theta)(R - G\cos\theta)}{\sin^3\theta} &= \frac{\sin\varphi}{I_y} L_y + \frac{\cos\varphi}{I_z} L_z + \dot{\theta}\Phi_{1\theta} + \dot{\psi}\Phi_{2\theta} + \dot{\psi}^2\Phi_{3\theta} \\ &+ \frac{\Delta R I^2 (I_x - 2I)(2\cos 2\varphi - \Delta)(G - R\cos\theta)}{4 I_x I_y I_z \sin\theta},\end{aligned} \tag{18}$$

$$\dot{R} = \frac{L_x}{I} - \frac{\Delta (G - R\cos\theta)^2 \sin 2\varphi}{2\sin^2\theta} + \dot{\theta}\Phi_{1R} + \dot{\theta}^2\Phi_{2R} + \dot{\psi}\Phi_{3R} + \dot{\psi}^2\Phi_{4R} + \dot{\theta}\dot{\psi}\Phi_{5R}, \tag{19}$$

$$\begin{aligned}\dot{G} &= \frac{\cos\theta}{I} L_x + \left(\frac{\sin\varphi}{I_z} L_z - \frac{\cos\varphi}{I_y} L_y \right) \sin\theta + \\ &+ \frac{\Delta (G - R\cos\theta)\sin 2\varphi}{2} \left(\frac{I^2 R (I_x - 2I)}{I_x I_y I_z} + \frac{(G - R\cos\theta)\cos\theta}{\sin^2\theta} \right) \\ &+ \dot{\theta}\Phi_{1G} + \dot{\theta}^2\Phi_{2G} + \dot{\psi}\Phi_{3G} + \dot{\psi}^2\Phi_{4G} + \dot{\theta}\dot{\psi}\Phi_{5G}.\end{aligned} \tag{20}$$

where $\Phi_i = \Phi_i(\nu, \psi, \gamma, \theta, \varphi, R, G)$ are functions that are given in appendix because of the bulkiness,

$$L_x = L_{Ix} - \frac{3\mu(I_y - I_z)\overline{r}_y \overline{r}_z}{r^3}, \quad L_y = L_{Iy} - \frac{3\mu(I_z - I_x)\overline{r}_x \overline{r}_z}{r^3}, \quad L_z = L_{Iz} - \frac{3\mu(I_x - I_y)\overline{r}_x \overline{r}_y}{r^3}.$$

D. Equations of the active spacecraft 3D motion

To obtain the equations of motion of an active spacecraft, we determine its position vector $\mathbf{r}_A$ in the inertial reference frame $OX_pY_pZ_p$

$$\mathbf{r}_A^p = \overrightarrow{OA} = \mathbf{r}^p + \boldsymbol{\rho}^p = \mathbf{r}^p + [\mathbf{M}_{pH}]\boldsymbol{\rho}^H, \tag{21}$$

where $\mathbf{r}^p$ is given by equation (5), $\boldsymbol{\rho}^H = [x_A, y_A, z_A]^T$ is the spacecraft's position vector in Hill coordinate frame $BX_HY_HZ_H$ (Fig. 1). After calculating the matrix product, the expression (21) is reduced to

$$\mathbf{r}_A^p = \begin{bmatrix} r\cos\nu\sin\psi + x_A\cos\psi - y_A\sin\nu\sin\psi + z_A\cos\nu\sin\psi \\ r\sin\nu + y_A\cos\nu + z_A\sin\nu \\ r\cos\nu\cos\psi - x_A\sin\psi - y_A\sin\nu\cos\psi + z_A\cos\nu\cos\psi \end{bmatrix}. \tag{22}$$

The Newton's law for the active spacecraft in the inertial reference frame $OX_pY_pZ_p$ has the form

$$m_A\ddot{\mathbf{r}}_A^p = \mathbf{G}_A^p + [\mathbf{M}_{pH}]\mathbf{P}^H, \tag{23}$$

where m_A is the mass of the spacecraft, $\mathbf{G}_A^p = -\frac{\mu m_A}{r_A^3}\mathbf{r}_A^p$ is the gravitational force, $r_A = \sqrt{(r+z_A)^2 + x_A^2 + y_A^2}$, $\mathbf{P}^H = [P_x, P_y, P_z]^T$ is the total thrust of the active spacecraft's engines. Substitution of vector (22) into (23) and expression of second derivatives $\ddot{x}_A$, $\ddot{y}_A$, $\ddot{z}_A$ from the obtained projections gives

$$\begin{aligned}
\ddot{x}_A &= -\frac{\mu x_A}{r_A^3} + \frac{P_x}{m_A} + (y_A \sin\nu - (r+z_A)\cos\nu)\ddot{\psi} + \dot{\psi}^2 x_A + 2\dot{\psi}\left((\dot{\nu} y_A - \dot{r} - \dot{z}_A)\cos\nu + ((r+z_A)\dot{\nu} + \dot{y}_A)\sin\nu\right), \\
\ddot{y}_A &= -\frac{\mu y_A}{r_A^3} + \frac{P_y}{m_A} - x_A\ddot{\psi}\sin\nu - (r+z_A)\ddot{\nu} + y_A\dot{\nu}^2 - 2\dot{x}_A\dot{\psi}\sin\nu - 2(\dot{z}_A + r)\dot{\nu} + (y_A \sin\nu - (r+z_A)\cos\nu)\dot{\psi}^2 \sin\nu, \\
\ddot{z}_A &= -\frac{\mu(r+z_A)}{r_A^3} + \frac{P_z}{m_A} - \ddot{r} + x_A\ddot{\psi}\cos\nu + y_A\ddot{\nu} + (r+z_A)\dot{\nu}^2 + 2\dot{y}_A\dot{\nu} + 2x_A\dot{\psi}\cos\nu \\
&\quad - (y_A \sin\nu - (r+z_A)\cos\nu)\dot{\psi}^2\cos\nu.
\end{aligned} \tag{24}$$

where x_A, y_A, z_A are coordinates of the spacecraft relative the space debris object in the Hill frame $BX_HY_HZ_H$. As can be seen from equations (24), the coordinates depend on the motion of point B, which is given by equations (6). Since the distance ρ is small compared to the radius r, discarding small terms in (24) gives

$$\begin{aligned}
\ddot{x}_A &= -\frac{\mu x_A}{r^3} + \frac{P_x}{m_A} + \dot{\psi}^2 x_A + \frac{\dot{\psi}\left(2(\dot{\nu} y_A - \dot{z}_A)\cos^2\nu + \dot{y}_A \sin 2\nu + 2\dot{\nu} y_A \sin^2\nu\right)}{\cos\nu} - \frac{F_{Ix}^H}{m_B}, \\
\ddot{y}_A &= -\frac{\mu y_A}{r^3} + \frac{P_y}{m_A} - \frac{F_{Iy}^H}{m_B} - 2\dot{x}_A\dot{\psi}\sin\nu + y_A\dot{\psi}^2\sin^2\nu + \dot{\nu}(\dot{\nu} y_A - 2\dot{z}_A) - \frac{2x_A\dot{\psi}\dot{\nu}\sin^2\nu}{\cos\nu}, \\
\ddot{z}_A &= -\frac{\mu z_A}{r^3} + \frac{P_z}{m_A} - \frac{F_{Iz}^H\cos\psi}{m_B} - \dot{\psi}^2 y_A \sin 2\nu + 2x_A\dot{\psi}\dot{\nu}\sin\nu + \dot{\psi}^2 z_A\cos^2\nu + 2\dot{x}_A\dot{\psi}\cos\nu + z_A\dot{\nu}^2 + 2\dot{y}_A\dot{\nu}.
\end{aligned} \tag{25}$$

The system of equations (6), (16)-(20) and (25) describes the motion of the considered mechanical system consisting of an active spacecraft and space debris.

E. Simplified model of a symmetrical space debris motion in GEO

A number of assumptions are made to obtain a simplified system of equations. It is supposed that the motion of mechanical system in GEO is considered. The influence of the gravity gradient torque is negligible, angular velocities $\dot{\psi}$, $\dot{\nu}$ are small, and the motion of the space debris object occurs under the influence of ion beam forces and torques only. It is assumed that the control system of the active spacecraft provides a constant relative position $x_A = d$, $y_A = z_A = 0$. It is also supposed that space debris is a symmetric rigid body. In this case $I_y = I_z = I$.

The assumptions made allow to write the equations of the space debris object attitude motion in the form

$$\ddot{\theta}+\frac{(G-R\cos\theta)(R-G\cos\theta)}{\sin^3\theta}=\frac{L_{Iz}^{\varphi}(\theta)}{I}\ , \tag{26}$$

$$\dot{R}=\frac{L_{Ix}^{\varphi}(\theta)}{I}\,, \tag{27}$$

$$\dot{G}=\frac{L_{Ix}^{\varphi}(\theta)\cos\theta-L_{Iy}^{\varphi}(\theta)\sin\theta}{I}\,, \tag{28}$$

$$\dot{\gamma}=\frac{G-R\cos\theta}{\sin^2\theta}\ , \tag{29}$$

$$\dot{\varphi}=\frac{R}{\overline{I}_x}-\frac{(G-R\cos\theta)\cos\theta}{\sin^2\theta}\ . \tag{30}$$

where $L_{Ix}^{\varphi}=L_{Ix}$, $L_{Iy}^{\varphi}=L_{Iy}\cos\varphi-L_{Iz}\sin\varphi$, $L_{Iz}^{\varphi}=L_{Iy}\sin\varphi+L_{Iz}\cos\varphi$ are projection of the ion beam torque on the coordinate system $BX_bY_3Z_2$, which is rotated relative to the Hill system at angles γ and θ. The ion beam torque projections L_{Ix}^{φ}, L_{Iy}^{φ}, L_{Iz}^{φ} at a fixed relative position of the active spacecraft and a fixed orientation of the ion beam axis depends only on the angle θ. To calculate these projections the technique described in [19] and in the third chapter of monograph [27] can be used. Calculations show that for a space debris object of cylindrical shape, in which the axis of symmetry is the BX_b axis, and the center of mass lies on this axis, projections L_{Ix}^{φ} , L_{Iy}^{φ} are equivalent to zero, and the projection L_{Iz}^{φ} is an odd function of θ angle. The function $L_{Iz}^{\varphi}(\theta)$ can be represented as a Fourier series

$$L_{Ix}^{\varphi}=L_{Iy}^{\varphi}=0\,,\qquad L_{Iz}^{\varphi}=uL_I^{\max}\sum_j^k b_j\sin j\theta\,, \tag{31}$$

where $u\in[0,1]$ is the control parameter, which is proportional to the square root of the ion flux velocity and can be controlled through the voltage inside the spacecraft's ion engine , $L_I^{\max}$ is the module of the ion beam torque amplitude value in the case when the engine is operating at full power, $b_j\in[-1,1]$ are expansion coefficients of the Fourier series.

Taking into account equation (31), the attitude motion of the cylindrical space debris object in GEO can be reduced to equations (29), (30), and

$$\ddot{\theta}+\frac{(G-R\cos\theta)(R-G\cos\theta)}{\sin^3\theta}=\frac{uL_I^{\max}}{I}\sum_j^k b_j\sin j\theta\,, \tag{32}$$

where R, and G are constants, which are determined by the initial angular velocities and the initial orientation of the space debris object. Equation (32) can be integrated independently of equations (29) and (30). The energy integral can be written for equation (32) in the form

$$\frac{\dot{\theta}^2}{2} + W(\theta) = E, \tag{33}$$

where E is constant, $W(\theta)$ is the reduced potential energy, which is determined by the expression

$$W(\theta) = \frac{G^2 + R^2 - 2GR\cos\theta}{2\sin^2\theta} + \frac{uL_I^{\max}}{I}\sum_j^k \frac{b_j}{j}\cos j\theta. \tag{34}$$

The view of function $W(\theta)$ defines the topology of the phase space of equation (32). Studying the motion of space debris of arbitrary shape seems to be a very difficult task due to the large range of values that can take the coefficients of the Fourier series b_j. Equation (34) can be used to search for stable equilibrium positions θ_* of equation (32), which correspond to the reduced potential energy $W(\theta)$ minima.

For comparison purposes, the case of planar oscillations will also be considered. It follows from [23] that oscillations of the space debris on GEO are determined by the equation

$$\ddot{\theta} = \frac{uL_I^{\max}}{I}\sum_j^k b_j \sin j\theta. \tag{35}$$

A comparison of equations (32) and (35) shows that in the planar case $G = R\cos\theta$.

III. Choosing a favorable attitude motion mode for contactless transportation

Previous studies [19, 23] performed for the planar motion case showed that the angular motion of space debris has a significant impact on transportation efficiency, which is expressed in the time required to an orbital maneuver. The process of contactless transportation in the case of planar motion is considered below schematically. It is assumed that there is a target trajectory of space debris calculated in some way and the corresponding dependence of the ion beam force modulus and direction. This trajectory and force will be called the program trajectory and the program force (Fig. 2a). When the space debris is blown by the ion stream, the magnitude and direction of the resultant force depends, among other things, on the orientation of the space debris in the stream (angle α on Fig. 2). The active spacecraft can be placed in such a position relative to space debris that the resultant of ionic forces will be directed along the required programmed force (Fig. 2b). The required force modulus can be obtained by changing the velocity of ions in the stream. Since the resultant ion beam force is applied not at the space debris center of mass, but at some other point K (Fig. 2b), a ion beam torque L_{iz} arises that tends to rotate the space debris object in the stream. This point will be called the center of ion pressure by analogy with the case of calculating aerodynamic forces. As a result of the action of the ion torque, the body rotates, which leads to a change in the magnitude and direction of the ion force (Fig. 2c). The existence of a force component $F_{i\perp}$ perpendicular to the program trajectory leads to the displacement of space debris from this trajectory (Fig. 2d). It is necessary to adjust the ion beam force in order to return the space debris to the program trajectory. To do this, move the spacecraft to a point corresponding to the adjusted ion beam force F_P' (Fig. 2e). Additional fuel will be spent on this maneuver.

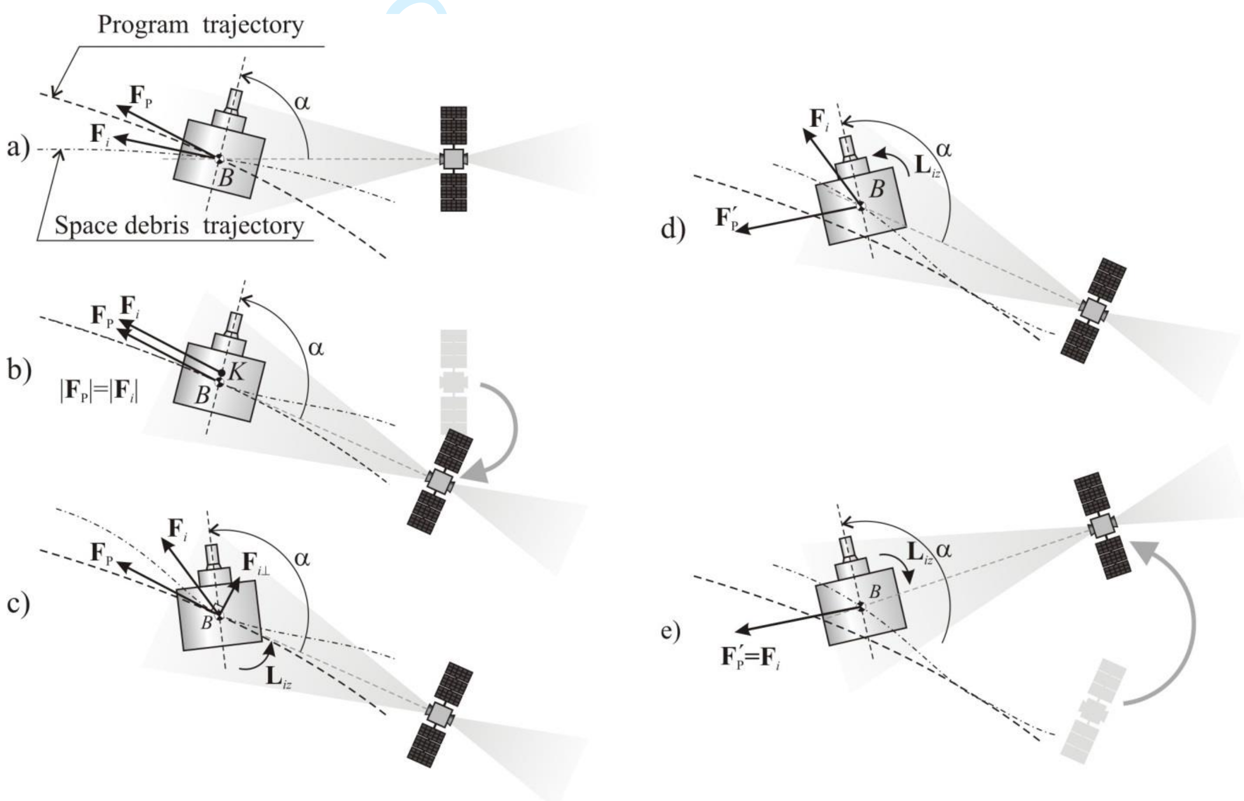

Fig. 2 Scheme of the active spacecraft control.

In order to avoid corrective flights of the active spacecraft and the corresponding fuel costs, space debris must be in a stable angular equilibrium position during contactless transportation. In this case, the ion flow blowing will not cause oscillations of the space debris and its displacement from the program trajectory.

If the period of oscillations of the space debris object is small with respect to the period of orbital flight, it is possible to use another approach to control, which is based on the use of averaged forces. The idea is to expose according to the program force not the instantaneous ion beam force, but the force averaged over the period of the space debris oscillations. In this case, the force component $F_{i\perp}$ perpendicular to the desired direction will be directed half period to one side and the other half to the opposite side. As a result, the trajectory of the space debris center of mass will oscillate around the program trajectory (Fig. 3). In this case, it is not necessary to spend fuel on moving the spacecraft to a new relative point to parry the perpendicular components of the ion beam force. As noted in [19], this method is suitable for transporting rapidly rotating objects.

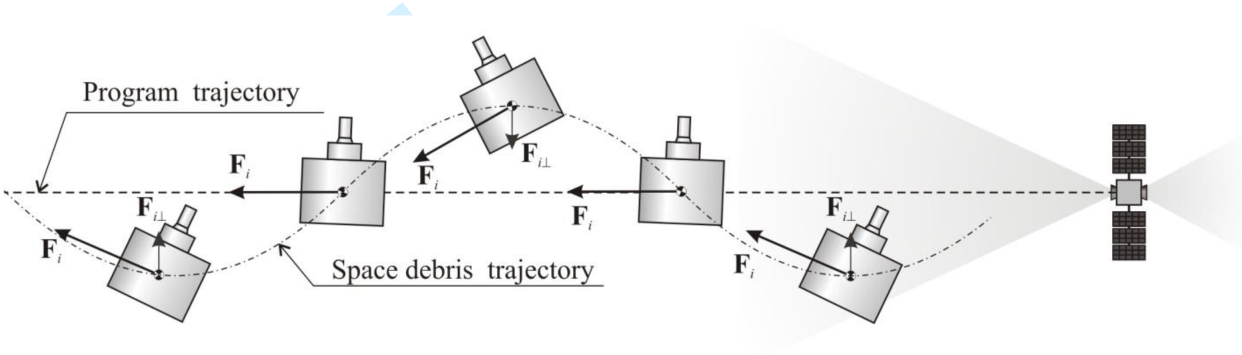

Fig. 3 Scheme of an averaged ion beam force control.

Comparing the transportation efficiency in the equilibrium mode and in the oscillation mode, it can be expected that the first method will be more efficient from the point of view of the ion engine fuel costs, since all the fuel is spent on useful work, in contrast to the case of oscillations, where only a part of the ion force is used for transportation. Since the magnitude and direction of the ion beam force in the equilibrium position depends on the shape of the space debris object, testing this hypothesis requires additional research that is beyond the scope of this work.

Consider the case of spatial motion. An analysis of equations (26) and (29) shows that transportation of space debris in the equilibrium position is possible only with certain combinations of parameters R, G and θ_*. If the point $\theta_* = \arccos(G/R)$ is the equilibrium position, then according to equation (26), the derivative of the angle is equal to zero. This condition is satisfied, when $L_{Iz}^{\varphi}(\arccos(G/R)) = 0$. The change of the function $L_{Iz}^{\varphi}(\theta)$ roots can be achieved by deflecting the ion beam axis from the AB line in the plane BX_HY_2. An analysis of this special case will be the subject of the following works of the authors. If $\theta_* \neq \arccos(G/R)$, the angular velocity $\gamma = \omega_\gamma = const$, and the space debris object is in regular precession mode, when the axis of the space debris moves along a cone. The ion beam force vector rotates with the object.

For simplicity, we restrict ourselves to considering the case when the program force is directed along the BX_H axis, and the spacecraft is located at the point with coordinates $(x_A, 0, 0)$ in the $BX_H Y_H Z_H$ frame. In the case of an axisymmetric object, the ion beam force vector $\mathbf{F}_i$ lies in plane $BX_H Y_2$ formed by the axis of space debris and the axis of the ion beam. In the case of regular precession, the projection of the vector $\mathbf{F}_i$ on the axis BX_H is constant ($F_{ix}^H = const$), and the projection on the axis BY_2, which is $F_{iy}^{(2)}$, will draw the circle in the plane $BY_H Z_H$ during the period of the angle γ change. Since the length of the projection $F_{iy}^{(2)}$ does not change due to the constancy of the angle θ, and $\dot{\gamma} = \omega_\gamma = const$, the average value F_{iy}^H over the period T_γ of γ oscillation will be zero

$$
\begin{aligned}
\bar{F}_{iy}^H &= \frac{1}{T_\gamma}\int_0^{T_\gamma} F_{iy}^H dt = \frac{1}{T_\gamma}\int_0^{T_\gamma} F_{iy}^{(2)} \cos\gamma dt = \frac{1}{T_\gamma} F_{iy}^{(2)} \int_0^{T_\gamma} \cos\left(\frac{2\pi}{T_\gamma} t\right) dt = 0, \\
\bar{F}_{iz}^H &= \frac{1}{T_\gamma}\int_0^{T_\gamma} F_{iz}^H dt = \frac{1}{T_\gamma}\int_0^{T_\gamma} F_{iy}^{(2)} \sin\gamma dt = 0.
\end{aligned} \quad (36)
$$

The equality of the average value $\bar{F}_{iy}^H$ to zero leads to the fact that in the case of a high precession rate ω_γ, there is no need to spend the addition fuel of the active spacecraft on parrying its departure from the program trajectory. In the case of arbitrary rotation of the space debris object, the average forces $\bar{F}_{ix}^H$, $\bar{F}_{iy}^H$ are not equal to zero, since $F_{iy}^{(2)} = F_{iy}^{(2)}(\theta) \neq const$, and $\gamma \neq \omega_\gamma t$. This means that the active spacecraft control system must constantly work to keep it near the program trajectory spending extra fuel on it.

Thus, transportation at a constant angle θ is most preferred mode. The constancy of the angle is observed either when the object is in equilibrium, or in the stationary mode of regular precession. In both cases, it is not necessary to spend additional fuel to parry the departure of the space debris center of mass from the program trajectory.

IV. Ion beam thrust control

Consider the thrust control of an ion engine, which provides the translation of the angle θ in the equilibrium position θ_*. The control law proposed in [24] for planer case is taken as a basis. A significant difference that required the processing of the control law is that the equilibrium position is nonzero.

$$u = \begin{cases} 1 + k(\theta - \theta_*)\dot{\theta}\,\mathrm{H}[(\theta_* - \theta)\dot{\theta}], & when\;\; k(\theta-\theta_*)\dot{\theta} > -1; \\ 1, & when\;\; k(\theta-\theta_*)\dot{\theta} \le -1; \end{cases} \tag{37}$$

where H is the Heaviside theta function, k is the constant control parameter, whish is chosen from the condition $u < 1$. The case $u < 0$ is not physically feasible, because it means an ion beam that is drawn into the engine of the spacecraft. It is proposed to initially take $k = 1$, and then choose this parameter based on data from the previous oscillation period T

$$. \qquad k = \frac{1}{\max\limits_{[t-T,t]}(|\,(\theta-\theta_*)\dot{\theta}\,|)}. \tag{38}$$

The results of numerical simulation presented in the next section confirm the possibility of using this control law to translate the angle θ to the equilibrium position θ_*.

V. Results of numerical simulations

As an example, the motion of an object close in its mass-geometric parameters to the Meteosat-8 [28] satellite in a GEO is studied below. The uncontrolled motion of the object and the controlled motion with law (37) are considered.

A. Space debris parameters

Motion simulation requires a preliminary calculation of ion beam forces and torques depending on the orientation of the object in the ion beam. To calculate them, a Matlab program was developed in accordance with the calculation procedure described in [19, 27]. The surface of the object was divided into triangles and the force effect of the ion flux on each of them was calculated. The geometric parameters of the considered object and an example of the computational mesh are shown in Fig. 4. The object and ion beam parameters needed to calculate the ion beam torque are given in Table 1. Fig. 5 shows the dependence of L_{Iz}^{φ} projection on the angle θ. The corresponding Fourier series expansion coefficients b_j are given in Table 2, and $L_I^{\max} = 3.706 \cdot 10^{-3}$ Nm.

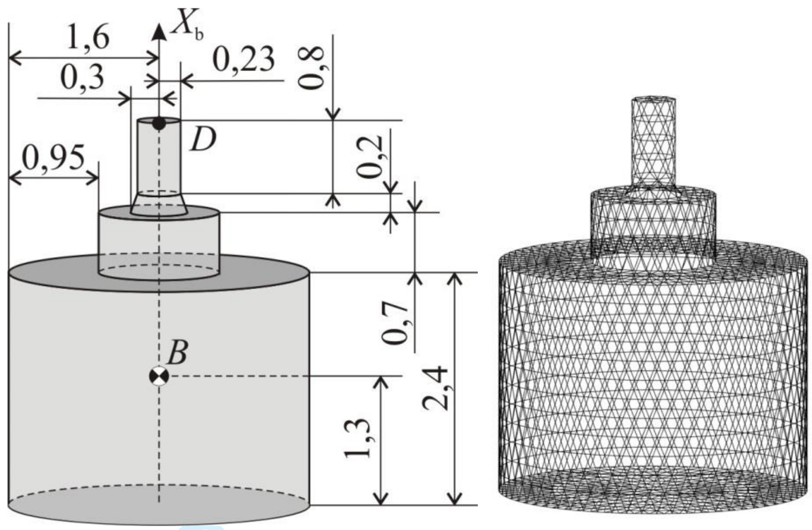

Fig. 4 Considered space debris object and calculation mesh.

Table 1 Space debris object's and ion beam parameters.

Parameter	Value
Total mass m_B	1100 kg
Principal moment on inertia I_x	1400 kg·m^2
Principal moments on inertia I_y , I_z	2100 kg·m^2
Plasma density	2.6·10^{16} m^{-3}
Mass of particle	2.18·10^{-25} kg
Radius of the beam at the beginning of the far region	0.1 m
Ion beam axial velocity	38 000 m/s
Ion beam divergence angle	15º
Distance *AB*	15m

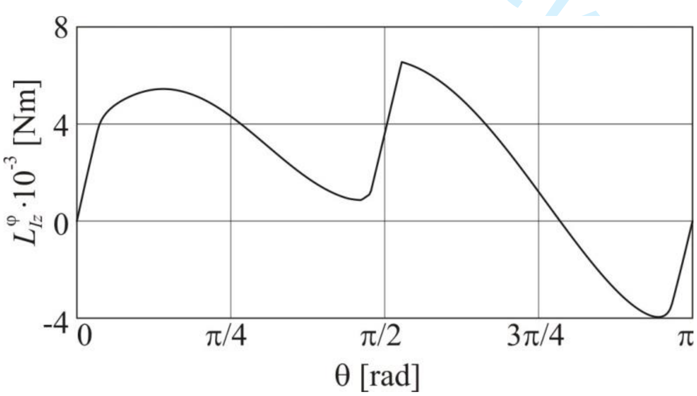

Fig. 5 Dependence of L_{Iz}^{φ} projection on the angle θ.

Table 2 Fourier series coefficients.

j	b_j	j	b_j	j	b_j	j	b_j
1	1	5	-0.0378	9	-0.0109	13	-0.0040
2	0.4482	6	0.0394	10	0.0076	14	-0.0066
3	-0.0002	7	-0.0304	11	-0.0083	15	-0.0013
4	0.8870	8	0.2792	12	0.1466	16	0.0800

The values given in Tables 1 and 2 can be used for numerical simulation of the space debris attitude motion.

B. Uncontrolled space debris attitude motion

For the coefficients given in Table 2, the reduced potential energy (34) has a single global minimum, which in the phase portrait corresponds to a stable equilibrium of the center type. The position of this center θ_* depends on the values of G and R, which in turn are determined by the initial angular velocities of space debris in accordance with (14). Fig. 6 shows the curves that determine the position of the center θ_* depending on the value of R for various G.

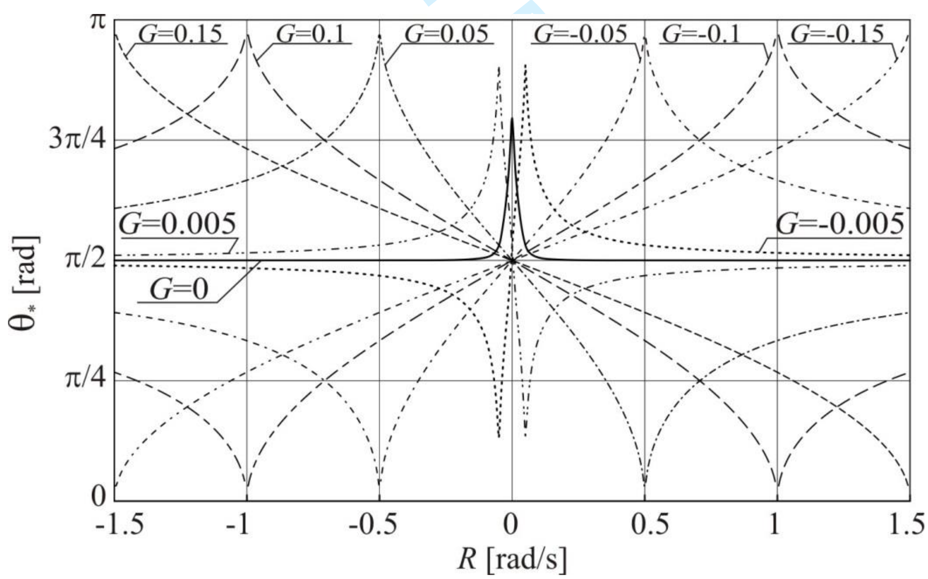

Fig. 6 Dependences of the equilibrium position   on the   for various.

Figure 7a shows the dependence of $W(\theta)$ for $G = 0.005\,\text{rad/s}$, $R = 0.01\,\text{rad/s}$. In this case to the equilibrium position is $\theta_* = 1.0597\,\text{rad}$. The corresponding phase portrait is shown in Figure 7b.

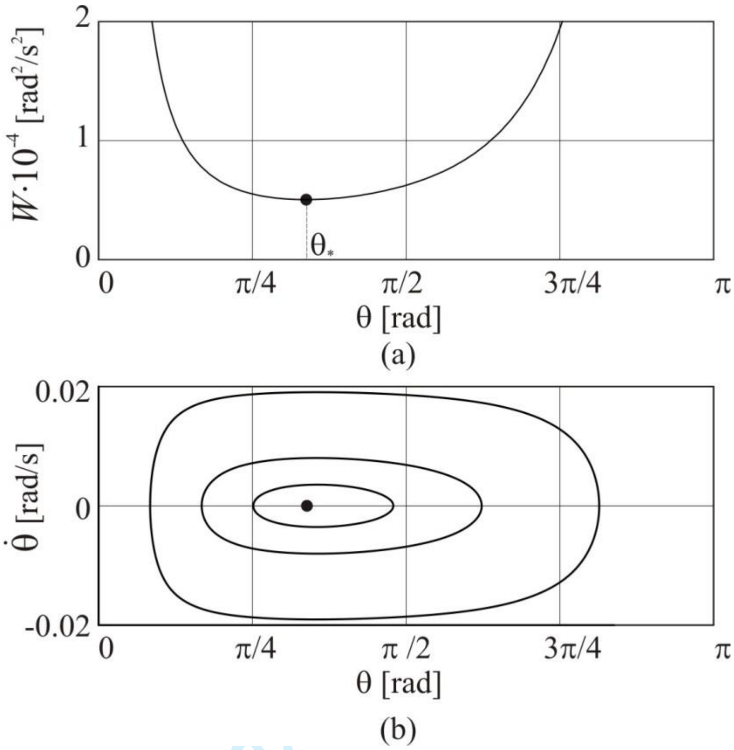

Fig. 7. Dependences of the reduced energy W on the angle θ and phase portrait for $G = 0.005\,\mathrm{rad/s}$, $R = 0.01\,\mathrm{rad/s}$.

In the case of planar motion, when $G = R\cos\theta$, a fundamentally different behavior is observed. The first term in expression (34) is equal to zero, and the $W(\theta)$ is 2π - periodic function. Moreover the area of definition of the angle θ is $[0,\ 2\pi]$. In this ideal case, the phase portrait contains both singular points of the center type, which correspond to the minima of the reduced potential energy, and points of the saddle type, which correspond to the maxima. Figure 8a shows the dependence of the reduced potential energy on angle θ. The corresponding phase portrait of the system is shown on Figure 8b. The black dots show the equilibrium points of the center type, and the white dots show equilibrium positions of the saddle type.

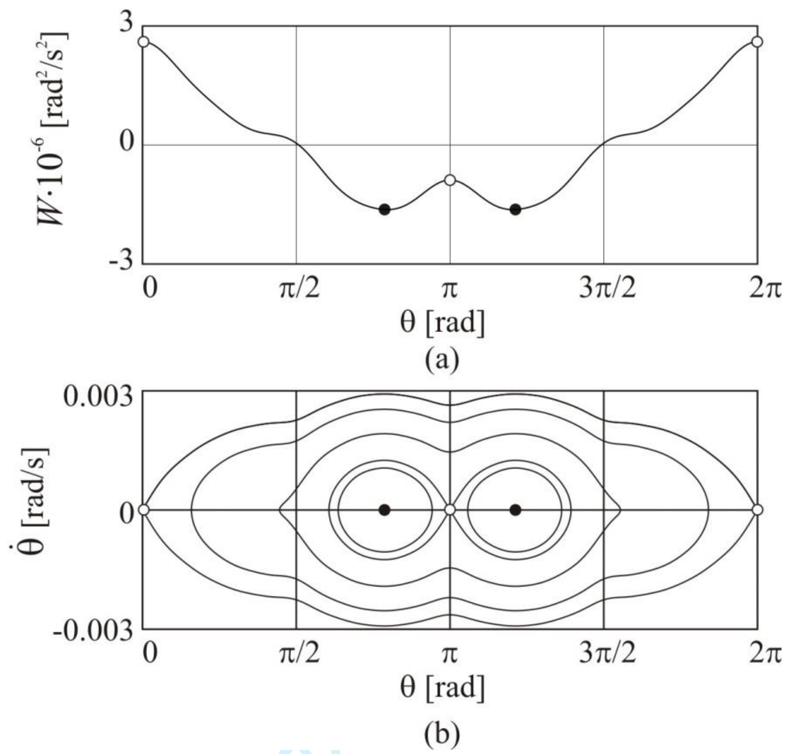

Fig. 8 Dependences of the reduced energy W on the angle θ and phase portrait for planar case when $G = 0$ and $R = 0$.

The fundamental difference between the planar case and the spatial case of the space debris motion is that finding the angle in the equilibrium position θ_*^p in the planar case means that the BX_b axis is stationary in the orbital coordinate system, while equilibrium of θ in the general case means that the axis will rotate along a cone with a constant angle θ_* (Fig. 9). This rotation is due to the constancy of the angular velocity $\dot{\gamma}$ in the case when $R \neq 0$, $G \neq 0$, $\theta = \theta_* = const$. Fig. 9 shows the trajectories of the points of intersection of the longitudinal axis BX_b with a sphere centered at point B in the Hill's coordinate system $BX_H Y_H Z_H$. Point D_1 corresponds to the motion in the planar case when $\theta = \theta_*^p$. This point is stationary on the sphere and has constant coordinates in $BX_H Y_H Z_H$. Point D_2 corresponds to the motion in the spatial case when $\theta = \theta_*$. A point moves in a circle on a sphere, and the center of this circle lies on the axis BX_H. A segment BD_2 rotates around the BX_H axis with a constant angular velocity $\dot{\gamma}$ defined by expression (29). In other words, the axis BX_b of the space debris object is in regular precession mode.

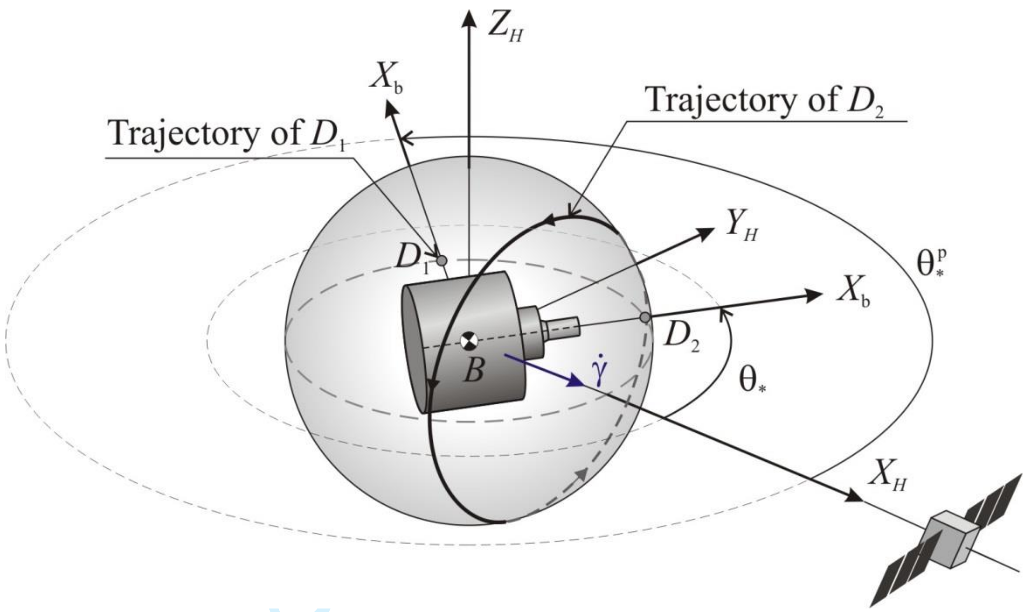

Fig. 9 Trajectories of the point of intersection of the longitudinal axis BX_b with a sphere centered at the space debris center of mass in planar (D_1 point) and general (D_2 point) cases.

C. Controlled space debris motion

As an example, consider the stabilization of the spatial motion of space debris using control (37), which at the initial moment of time has parameters: $\omega_x = 0.001\,\text{rad/s}$, $\omega_y = \omega_z = 0$, $\theta_0 = 2\,\text{rad}$, $\gamma_0 = \varphi_0 = 0$, $\dot{\theta}_0 = 0.001\,\text{rad/s}$ for which $R = 6.6667\cdot10^{-4}$ rad/s, $G = -2.7743\cdot10^{-4}$ rad/s, $\theta_* = 2.4082\,\text{rad}$. In the case of uncontrolled motion of space debris, the axis of the spacecraft makes the precession and nutation oscillations, shown in Fig. 10a. The control law (37) allows suppressing nutational fluctuations and reducing the motion to a regular precession Fig. 10b. Fig. 11 shows the change in angle θ in the case of uncontrolled motion and when using the control law (37). The corresponding graphs for the angle γ are shown in Fig. 12.

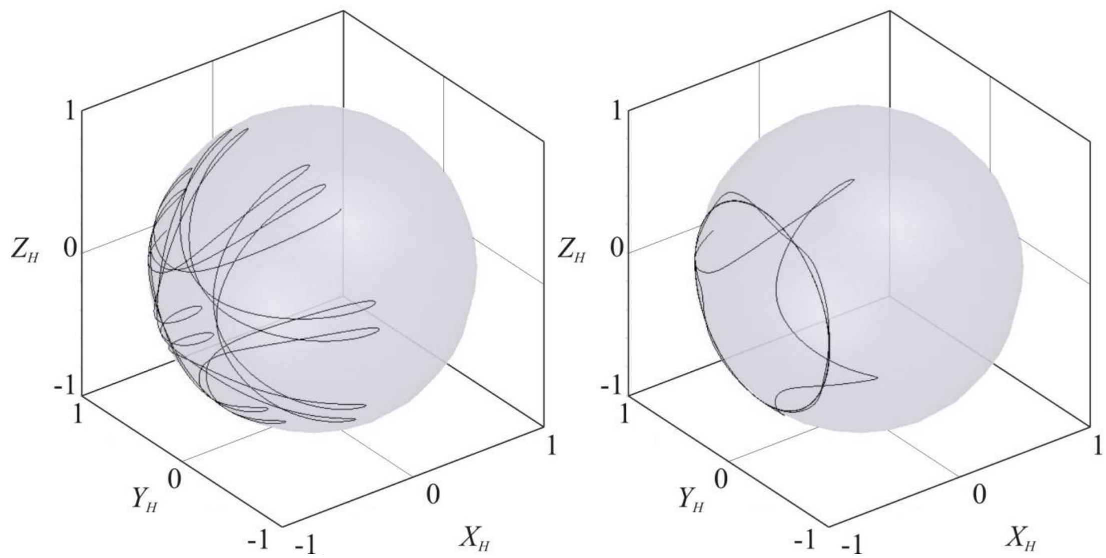

Fig. 10 Trace from the axis BX_b on the sphere centered at point B in uncontrolled (a) and controlled (b) cases.

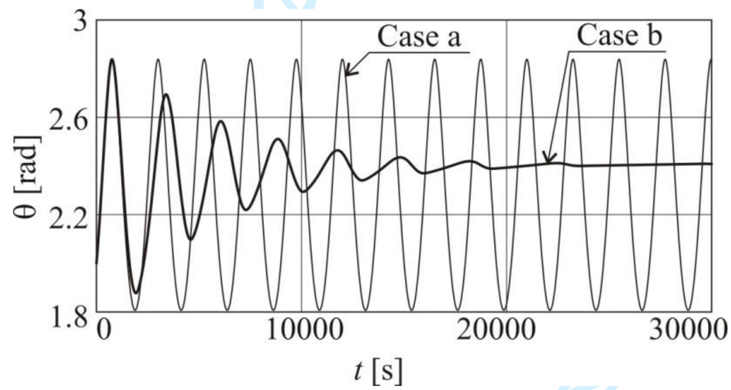

Fig. 11 The dependence of the angle θ on time in the case of the uncontrolled motion and when using the control law (37).

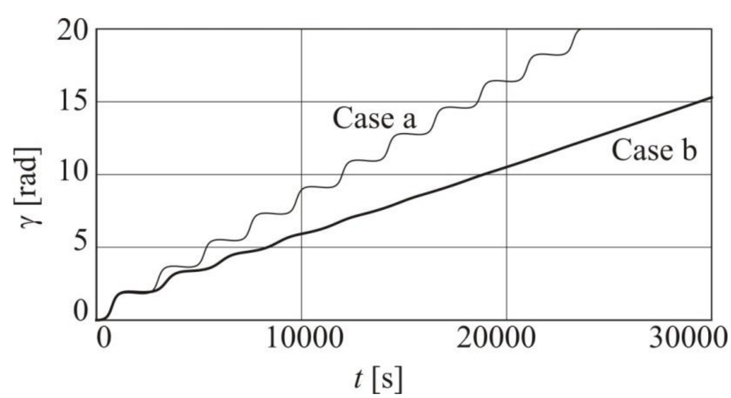

Fig. 12 The dependence of the angle γ on time in the case of the uncontrolled motion and when using the control law (37).

Thus, the proposed control law makes it possible to stabilize the angle fluctuations and put the motion of the axis of the space debris in the regular precession mode.

VI. Conclusion

The spatial motion of space debris during its contactless transport by an ion beam, which is created by an active spacecraft, was considered in this study. A mathematical model describing the three-dimensional motion of a mechanical system consisting of the space debris object and the active spacecraft under the influence of gravitational forces and torques, as well as forces and moments of the engines of the spacecraft, was developed. A simplified mathematical model describing the motion of symmetrical space debris in GEO with the fixed relative position of the active spacecraft was constructed. An analysis of the simplified equations of motion was carried out and various transport schemes were considered. It was shown that the transportation of space debris when it is in a state of equilibrium or in regular precession mode avoids the additional fuel costs of an active spacecraft. The law of controlling the velocity of ions in the ion beam, which provides stabilization of the angle between the axis of symmetry of the symmetrical space debris object and the line connecting the center of mass of space debris and the active spacecraft in GEO, was proposed. A numerical simulation of the motion of space debris close in shape to the Meteosat-8 satellite was carried out. Influence of the initial angular velocities of the space debris object on the arrangement of equilibrium positions was studied. The results of numerical simulations confirm the effectiveness of the proposed law. In contrast to the case of planar motion, the proposed control law does not translate the space debris into a stationary relative state, but into a regular precession mode, in which the angle θ is constant, and angle γ grows linearly. The results of the work showed that control laws developed for the planar case can be useful for the case of 3D motion.

Appendix

The angular velocity coefficients for the system of equations (18)-(20) are given here

$$\Phi_{1\theta} = \frac{\Delta I^2 R(2I - I_x)\sin 2\varphi}{2I_x I_y I_z}, \qquad \Phi_{2\theta} = \frac{\Delta I^2 R(2I - I_x)\sin 2\varphi \sin(\nu - \gamma)}{2I_x I_y I_z} + \frac{2(G - R\cos\theta)\cos(\nu - \gamma)}{\sin^2\theta},$$

$$\Phi_{3\theta} = \frac{\cos\theta\cos^2(\nu-\gamma)}{\sin\theta}, \quad \Phi_{1R} = \frac{\Delta(G-R\cos\theta)\cos 2\varphi}{\sin\theta}, \quad \Phi_{2R} = -\frac{\Delta}{2}\sin 2\varphi,$$

$$\Phi_{3R} = \frac{\Delta(G-R\cos\theta)\sin(\nu-\gamma)\cos 2\varphi}{\sin\theta}, \quad \Phi_{4R} = -\frac{\Delta}{2}\sin^2(\nu-\gamma)\sin 2\varphi,$$

$$\Phi_{5R} = -\Delta\sin 2\varphi\sin(\nu-\gamma),$$

$$\Phi_{1G} = \frac{RI\sin\theta}{I_x} + \frac{\Delta(G-R\cos\theta)\cos\theta\cos 2\varphi}{\sin\theta} + \frac{\Delta RI^2(\Delta+2\cos 2\varphi)\sin\theta}{4I_yI_z} - \frac{RI^3(\Delta^2+4\Delta\cos 2\varphi+4)\sin\theta}{4I_xI_yI_z},,$$

$$\Phi_{2G} = -\frac{\Delta}{2}\cos\theta\sin 2\varphi,$$

$$\Phi_{3G} = \frac{(\Delta\cos 2\varphi-1)(G-R\cos\theta)\sin(\nu-\gamma)\cos\theta}{\sin\theta} - \frac{I^2R(\Delta(2I-I_x)\cos 2\varphi+(\Delta^2I-2I_x))\sin(\nu-\gamma)\sin\theta}{2I_xI_yI_z},,$$

$$\Phi_{4G} = \frac{\Delta\sin 2\varphi\cos\theta}{2}\left(\cos 2\nu\cos^2\gamma-\cos^2\nu+\frac{1}{2}\sin 2\nu\sin 2\gamma\right) - \frac{\sin(2(\nu-\gamma))}{2},$$

$$\Phi_{5G} = -\Delta\sin 2\varphi\cos\theta\sin(\nu-\gamma)-\cos(\nu-\gamma).$$

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