Debris Removal in GEO by Heavy Orbital Collector

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Abstract. A new way to removal small space debris using a heavy collector in GEO is considered. The proposed scenario involves three types of consecutive maneuvers: capturing of debris into an area bounded by Hill sphere of the heavy collector, towing and discharging debris in a graveyard orbit. The plane motion equations of the debris relative to the collector are written in the Local-Vertical-Local-Horizontal frame taking into account the engine thrust of the collector. Also considering the engine thrust, the equation motion of the collector in oscillating elements is written. The interaction of the debris and the collector is described by means of the inelastic impact theory. The values of the thrust force of the collector are determined, at which the debris can be captured and towed or discharged in the graveyard orbit. It has been shown numerical simulation, that the proposed maneuvers can be implemented.

Keywords: Geostationary orbit; Space debris removal; Heavy collector; Artificial Lagrange points

1. Introduction

Currently, more than 900 communications and weather satellites are in Geostationary Orbit (GEO) and their total mass is about 2,500 metric tons [1]. The collisions of large space debris with other debris can significantly increase the number of small debris objects on the Earth orbit. Several orbits can be dangerous for new missions, and therefore space debris should be removed. On February 10, 2009, an inactive Russian communications satellite, designated Cosmos 2251, collided with an active commercial communications satellite operated by US-based Iridium 33 at a relative velocity of about 10 km/s, and at almost right angles to each other. This collision produced almost 2,000 pieces of debris, measuring at least ten centimeters in diameter, and many thousands more smaller pieces [2]. The choice of the active debris removal technique depends on properties of space debris. Recently several active debris removal methods have been developed [3-12]. There are three types of the connection between a space tug and space debris: the flexible connection, the rigid connection and the distant interaction. The act of docking onto such large and tumbling space objects is very challenging, and as a result novel touchless debris removal or despinning solutions are being explored. The ion-shepherd method uses the ion engine exhaust to

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push and/or despin a satellite [8-9], while the laser ablation method uses the debris' own mass as a thruster fuel source [10]. A promising touchless and low-power solution is the electrostatic tractor [11-12]. A robot arm can realize rigid connection between the space tug and the debris. A tether attached to the space debris can provide the flexible connection. The active debris removal using the space tug with a tether is one of promising techniques [3-7]. The contactless methods to remove space debris through ion beam or electrostatic forces do not require large amounts of fuel, but requires electrical energy, which can be obtained using, for example, solar panels. Therefore, these contactless methods have an indisputable advantage to removing space debris from GEO. However, this applies only to intact non-functioning satellites or large pieces of debris. Nevertheless, what to do with removal a lot of small debris that can occur as a result of potential collisions of large space debris.

The paper [13] proposes an original method of removing small debris objects from GEO, which is based on the nature of the libration motion in the three-body problem, similar in the Earth-Moon problem, when centrifugal inertia forces and Coriolis forces in the Earth-Moon system and gravitational forces from the Earth and the Moon act on a third small body. In our task, a heavy satellite as orbital collector located near GEO plays the role of a Moon.

The aim of this paper is to rationale the new way of capturing and removal small debris objects as a result of a collision between satellites. A key focus is to investigate the possibility of implementing the scenario, which involves three types of consecutive maneuvers: capturing of debris into an area bounded by the Hill sphere of the collector, towing and discharging debris in the graveyard orbit. A mathematical model to simulate planar motion of the collector relative to the Earth has been developed to substantiate and to study these maneuvers. The shape of the collector section in the plane of motion of the Earth-Collector system is assumed as an inelastic disk. The proposed mathematical model allows, in the first place, to take into account, in addition to the centrifugal forces of inertia, Coriolis forces and gravitational forces, also a thrust force of the collector and, secondly, to take consider an impact of small debris objects with the collector according to the laws of the inelastic impact theory. We show that all of these maneuvers are completely realizable, and in addition, we determine the values of the thrust force of the collector, at which the debris can be captured and towed or discharged in the graveyard orbit. All basic ideas of the proposed approach are confirmed by the direct numerical integration of the motion equations.

The paper consists of eight sections and conclusion. In the Section 1 (Introduction) the research objective is formulated. In the Section 2 the main idea of the proposed method is described. The Section 3 the motion equations of the debris as a material point relative to the collector are written in the classical form of the restricted three-body problem (Earth-Collector-Debris). The motion described by these equations are called undisturbed and the Jacobi integral corresponds to them. In the Section 4 all key assumptions are given, and the motion equations of the debris relative to the collector are written in the Local-Vertical-Local-Horizontal frame taking into account the engine thrust of the collector. Also, Gaussian variational equations of motion are used to a required time and a propellant mass needed for the GEO–GO transfer of the collector. The Section 5 shows several numerical examples of the capture and towing of the debris by the collector, taking into account the engine thrust and an impact theory of the interaction of the debris with the collector. The Section 6 investigates the influence of the engine thrust and an angular velocity of the rotation collector on discarding collected the debris in the graveyard orbit. In the section 7 requirements for the collector configuration and its attitude motion are formulated. At last, the conclusions together with the discussion are presented in the Section 8 (Conclusions).

2. Basic idea of the method

In our task, a heavy satellite (orbital collector) located near GEO plays the role of the Moon. Assume that two point masses (the Earth and the orbital collector), called as the primaries, revolve around their center mass in circular orbit close to GEO. A third small object (small space debris) moves in the plane of the primaries motion. Let us answer the question of what should be the mass of the collector, so that a Hill sphere radius is not too small. The Hill sphere of an astronomical body is the region in which it dominates the attraction of another astronomical body. The outer shell of that region constitutes a zero-velocity surface called as the Hill sphere. To be retained by the collector, the debris must have an orbit that lies within the collector's Hill sphere. Here the radius of the Hill sphere is defined as the half-distance between Lagrangian points and (Fig. 1). The Hill sphere radius is given by the well-known formula [14]

$$r_H \approx r\left(\frac{\mu}{3}\right)^{\frac{1}{3}} \qquad \left[\mu = m_1/\left(m_1+m_3\right) << 1\right], \tag{1}$$

where r is the distance between the Earth and the collector, m_3 is the mass of the Earth, m_1 is the mass of the collector. The equation (1) shows that for the collector mass of 100 metric tons, the Hill sphere radius is equal 7.5 meters.

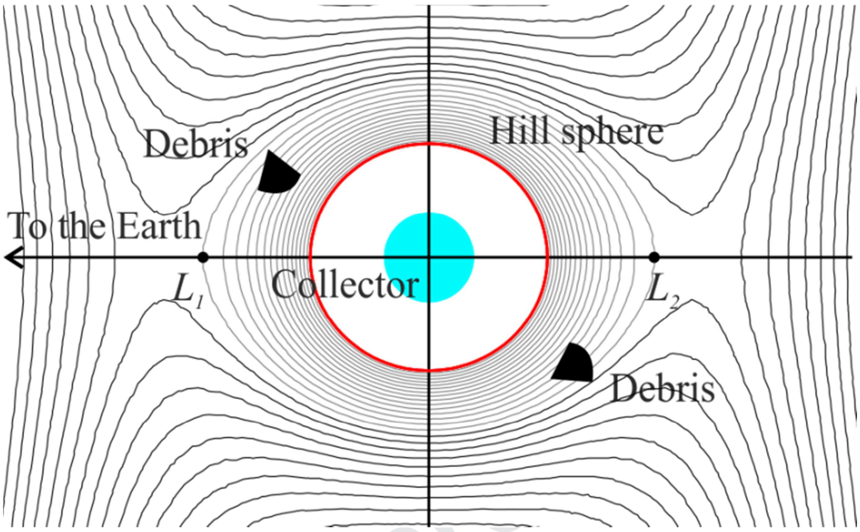


Fig. 1. The Hill sphere and artificial Lagrangian points L_1 and L_2.

The basic idea of the method is to move the collector to place into such position, that the debris will be inside the Hill sphere. Then the collector with the attracted debris system can move to graveyard orbit (GO). After the transportation of the system into a graveyard orbit, the collector must be unloaded, and the debris starts its own independent motion.

Thus, the scenario of the proposed mission involves the following four phases (Fig. 2):

- The capture of small debris objects one by one by the collector.
- Towing of the small debris objects located within the Hill sphere by the collector.
- Unloading collected swarm of the debris in the graveyard orbit.
- Returning the collector to GEO for next debris removal missions.

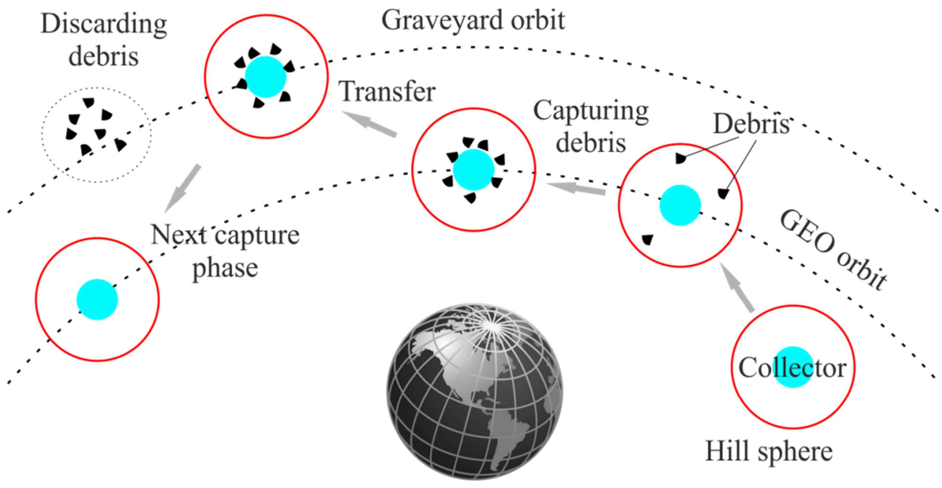


Fig. 2. Three types of the maneuvers of the debris removal mission: the capture, towing and discharging of the debris in the graveyard orbit.

3. Equations of motion in the classical form of the restricted three-body problem

To illustrate the behavior of the debris in the neighborhood of the collector, we can use the classical equations in Cartesian coordinates [15, 16]

$$\ddot{x} = \frac{\partial W}{\partial x} + n^2 x + 2n\dot{y}, \quad \ddot{y} = \frac{\partial W}{\partial y} + n^2 y - 2n\dot{x} \tag{2}$$

$$W(x, y) = \frac{Gm_1}{\sqrt{\left(x - r(1-\mu)\right)^2 + y^2}} + \frac{Gm_3}{\sqrt{\left(x + r\mu\right)^2 + y^2}}, \tag{3}$$

where $r = const$ is the distance between the Earth and the collector, $G = 6.67428 \cdot 10^{-11}\ \mathrm{m^3 s^{-2} kg^{-1}}$ is the universal gravitational constant, $n = 2\pi / \mathrm{T}$ is the main motion, (x, y) are coordinates of the debris in the Earth-Collector frame Oxy (Fig. 3).

The Jacobi integral corresponds motion to Eqs. (2) [14, 15]

$$J = 2G\left(\frac{m_1}{\sqrt{\left(x - r(1-\mu)\right)^2 + y^2}} + \frac{m_3}{\sqrt{\left(x + r\mu\right)^2 + y^2}}\right) + n^2\left(x^2 + y^2\right) - \left(\dot{x}^2 + \dot{y}^2\right) = const, \tag{4}$$

where J is the negative doubled total energy per unit mass in the rotating Cartesian frame Oxy (Fig. 3). The first term corresponds to the centrifugal potential energy, the second term represents gravitational potential and the third - is the kinetic energy. The forces that act on the debris are the two gravitational attractions, the centrifugal force and the Coriolis force. Since the first three can be derived from potentials and the last one is perpendicular to the trajectory, they are all conservative. Therefore the energy conserves its constant value.

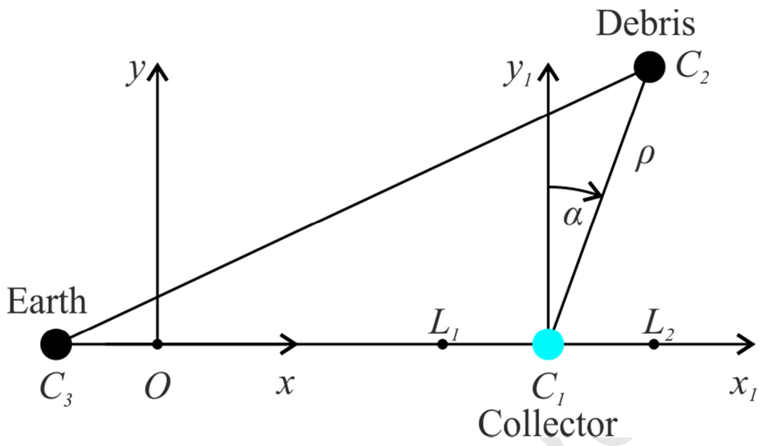


Fig. 3. The Earth-Collector frames.

Fig. 3 shows the debris position relative to the collector in the Cartesian coordinates $C_1x_1y_1$

$$\mathbf{r} = \overline{C_1C_2} = \left(\rho\sin\alpha, \rho\cos\alpha\right). \tag{5}$$

It is possible to determine the distance between the collector and the debris, if we use the polar reference frame with the change of variables

$$x = (1-\mu)r + \rho\sin\alpha, \quad y = \rho\cos\alpha. \tag{6}$$

In the polar coordinates (ρ, α), Eqs. (2), (3) are rewritten as

$$\ddot{\rho} = -r(\mu-1)n^2\sin\alpha - \frac{fm_1}{\rho^2} - \frac{fm_3\left(r\sin\alpha+\rho\right)}{\left(r^2+2r\rho\sin\alpha+\rho^2\right)^{3/2}} + \rho\left(n-\dot{\alpha}\right)^2, \tag{7}$$

$$\rho\ddot{\alpha} = -\frac{fm_3 r\cos\alpha}{\left(r^2+2r\rho\sin\alpha+\rho^2\right)^{3/2}} + r\left(1-\mu\right)n^2\cos\alpha + 2\left(n-\dot{\alpha}\right)\dot{\rho}, \tag{8}$$

then the Jacobi integral (4) has the form

$$J = r^2(1-\mu)^2n^2 + 2r(1-\mu)n^2\rho\sin\alpha + 2G\left(\frac{m_1}{\rho} + \frac{m_3}{\sqrt{r^2+2r\rho\sin\alpha+\rho^2}}\right) + \rho^2\left(n^2-\dot{\alpha}^2\right) - \dot{\rho}^2 \tag{9}$$

To the numerical simulation of the behavior of the debris in the neighborhood of the collector with mass m_1=100000 kg we can take the following initial conditions for the collector motion Eqs. (7) and (8):

$$\rho_0 = 7.4m, \qquad \dot{\rho}_0 = 0, \alpha_0 = 0, \dot{\alpha}_0 = \frac{\pi}{2}. \tag{10}$$

Fig. 4 shows the debris trajectories relative to the collector in the Cartesian coordinates $C_1 x_1 y_1$ (Fig. 3)

$$\mathbf{r} = \overline{C_1 C_2} = (x_1, y_1) = (\rho \sin\alpha, \rho\cos\alpha). \tag{11}$$

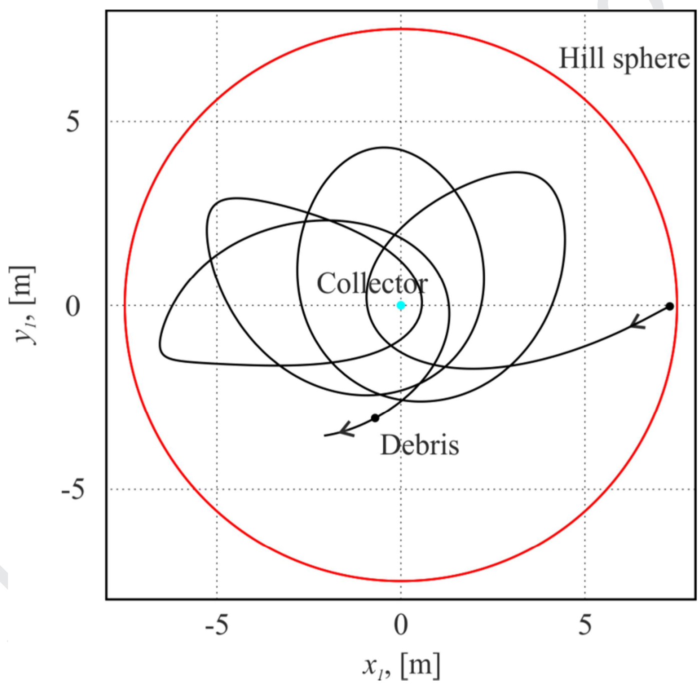


Fig. 4. The debris trajectories relative to the collector in the Cartesian coordinates (11) at the initial conditions (10).

The trajectory of the conservative system (7), (8) starting inside the Hill sphere, will never leave the boundaries of this sphere.

4. Motion Equations describing dynamic maneuvers of the heavy collector

4.1. Key assumptions

To derive the basic equations we introduce the following assumptions:

1. Mass of the small space debris object is significantly less than mass of the heavy collector

$$m_2 = m_1 . \tag{12}$$

2. In all considered cases only in-plane motion is studied.
3. The shape of the collector section in the plane of motion of the Earth-Collector system is assumed as an inelastic disk.
4. The engine thrust force P is directed along the tangent to the trajectory.

4.2. Relative motion with respect to a constantly accelerating frame

The implementation of all three types of maneuvers of the mission for the capture of the debris in GEO, towing debris and its subsequent unloading in the graveyard orbit is possible only with the use of the thrust force of the collector. In addition to the forces of inertia and the Coriolis force, the thrust force acts on the collector. In this case Eqs. (7) and (8) are not suitable, but for the classical Clohessy-Wiltshire equations [15] of the relative motion are quite applicable. Let us consider the motion of the debris relative to the local-vertical-local-horizontal (LVLH) or the Euler–Hill frame $C_1 x_1 y_1$ [17] (Fig. 5). The rectilinear LVLH frame is attached to the collector. The position of the debris relative to the space tug is described by the vector

$$\boldsymbol{\rho} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} . \tag{13}$$

The LVLH $C_1 x_1 y_1$ is not inertial, so the equations of the debris relative to the space tug contains the terms associated with the motion of the LVLH frame relative to the Earth. The classical Clohessy-Wiltshire equations are written as [15]

$$\ddot{x}_1 - 2n\dot{y}_1 - 3n^2 x_1 = a_x , \tag{14}$$

$$\ddot{y}_1 + 2n(t)\dot{x}_1 = a_y , \tag{15}$$

where n is the orbital rate of the space tug which is changed under the action of the tug's thrust. The change of the orbital rate $\dot{n}$ is approximated as

$$\dot{n} = \frac{P}{m_1 + m_2} \frac{1}{r} , \tag{16}$$

where r is the distance from the Earth center to the mass center of the collector. We can neglect the term $\dot{n}$ due to its smallness [18].

The right part of the Eqs. (14) and (15) includes projections of the acceleration

$$\mathbf{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix} = -\frac{G\left(m_1 + m_2\right)}{\rho^3}\boldsymbol{\rho} - \frac{P}{m_1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{17}$$

where P is the tangential thrust, $\mathbf{a}$ is the acceleration produced by the engine thruster of the collector $\mathbf{P}$ and by the gravitational force

$$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{Gm_1m_2}{\rho^3}\boldsymbol{\rho}. \tag{18}$$

Let us rewrite Eqs. (14) and (15) using polar coordinates ρ and α that will be more suitable to use during the analysis of the relative motion. After substituting

$$x_1 = \rho\sin\alpha, \qquad y_1 = \rho\cos\alpha \tag{19}$$

into Eqs. (14) and (15), we obtain

$$\ddot{\rho} = -\frac{P}{m_1}\cos\alpha - \frac{G\left(m_1 + m_2\right)}{\rho^2} + \rho\left(3n^2\sin^2\alpha - 2n\dot{\alpha} + \dot{\alpha}^2\right), \tag{20}$$

$$\ddot{\alpha} = \frac{3}{2}n^2\sin 2\alpha + 2\frac{n - \dot{\alpha}}{\rho}\dot{\rho} + \frac{P}{m_1\rho}\sin\alpha, \tag{21}$$

$$\dot{n} = \frac{P}{m_1 + m_2}\frac{1}{r}. \tag{22}$$

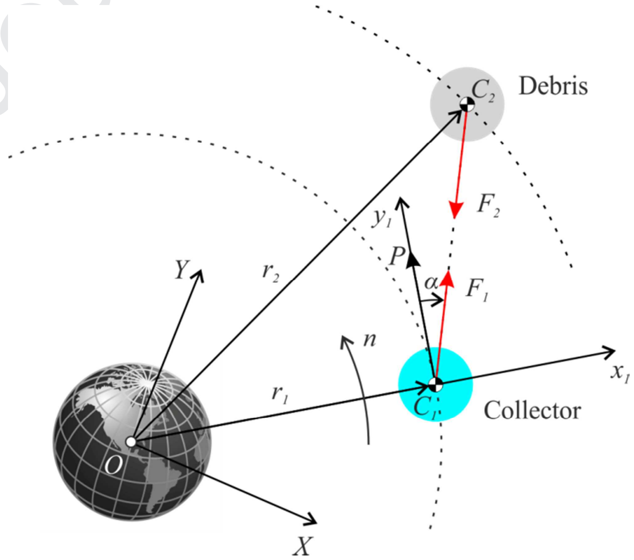


Fig. 5. The Illustration of the Collector-Debris LVLH frame.

4.3. Coplanar transfer from GEO to a circle graveyard orbit

The electric-propulsion thrust force is so low, that a continuous-thrust maneuver follows an unwinding spiral trajectory when starting from a circular orbit. The “quasi-circular” nature of the low-thrust spiral transfer allows us to use the Gauss variational equations in the inertial frame OXY (Fig. 5 for semimajor axis a, eccentricity e, and true anomaly θ in the following form [19]

$$\dot{a} = 2\frac{a^2 v}{\mu_E} a_t , \tag{23}$$

$$\dot{e} = 2\frac{e+\cos\theta}{v} a_t , \tag{24}$$

$$\dot{\theta} = \frac{h}{r^2} - 2\frac{\sin\theta}{ev} a_t , \tag{25}$$

where $h = \sqrt{\mu_E p}$ is angular momentum, $p = a\left(1-e^2\right)$ is parameter of the orbit, $\mu_E = Gm_3$ is the gravitational parameter of the Earth, $r = p/\left(1+e\cos\theta\right)$ is the distance -between the center of mass of the collector and the center of the Earth. Tangential thrust acceleration is $a_t = P/m_1$ and the orbital velocity is determined by

$$v = \sqrt{\frac{\mu_E}{r}} . \tag{26}$$

Let's find fuel consumption and transfer time required for the delivery of debris from GEO to the graveyard orbit. Assume that the diminishing the collector mass is

$$m_1 = m_{10} - \dot{m}t , \tag{27}$$

where $\dot{m} = P/c$ is the mass-flow rate of the low-thrust engine of the collector, c is an effective jet exhaust velocity.

Then the time required to complete the continuous-thrust GEO–GO transfer is obtained by integration of Eqs. (23)-(25) [19]

$$t_f = \frac{m_{10}}{\dot{m}}\left[1-\exp\left(\frac{\Delta V}{c}\right)\right]. \tag{28}$$

The difference between the initial and final circular velocities is

$$\Delta V = \sqrt{\frac{\mu_E}{r_{GEO}}} - \sqrt{\frac{\mu_E}{r_{GO}}}\ , \tag{29}$$

where r_{GEO} is the radius of GEO, r_{GO} is the radius of GO.

Finally, the propellant mass needed for the GEO–GO transfer of the heavy collector with the small debris objects can be estimated as

$$\Delta m = t_f \dot{m}. \tag{30}$$

Compute the low-thrust transfer time and propellant mass for a coplanar transfer from GEO to graveyard orbit, which is 200 km above GEO. The initial mass of the collector with the small debris objects in GEO is 100,000 kg. The orbital velocities for these orbits, respectively, are

$$V_{GEO} = \sqrt{\frac{\mu_E}{r_{GEO}}} = 3.075\, km/s\ ,\ \ V_{GO} = \sqrt{\frac{\mu_E}{r_{GO}}} = 3.067\, km/s\ . \tag{31}$$

Ion thrusters produce a total $1.0\,N$ of thrust, and each engine has the effective jet exhaust velocity $16{,}000\, m/s$. We use Eqs. (28) and (30) to compute the time and the propellant mass required to complete the continuous-thrust GEO–GO transfer:

$$t_f = 8.41\,\text{days} \qquad \Delta m = 45.4\,\text{kg}\ . \tag{32}$$

If the total thrust is equal $0.04\,N$, the propellant mass will not change, and the transfer time will be

$$t_f = 210.2\,\text{days} \qquad \Delta m = 45.4\,\text{kg}\ . \tag{33}$$

6. The capture and towing of the debris by the collector

In this section, using a numerical simulation, we illustrate the capture of the debris into the Hill sphere of the collector. Firstly, consider the case at the absence of the engine thrust

$$P = 0\ . \tag{34}$$

If the collector mass is equal to 100 tons, then the Hill sphere has the radius 7.5 meters. The initial conditions for the motion of the debris relative to the collector are chosen from the Hill sphere $(\rho_0 < r_H)$

$$\rho_0 = 7.4\,m,\ \dot{\rho}_0 = 0,\ \alpha_0 = \frac{\pi}{2},\ \dot{\alpha}_0 = 0\ , \tag{35}$$

then the debris does not leave the Hill sphere (Fig. 6a). If at the initial moment the debris were placed outside the Hill sphere $(\rho_0 > r_H)$

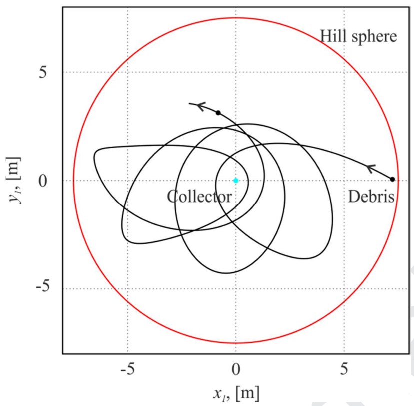
$$\rho_0 = 7.6\,m,\ \dot{\rho}_0 = 0,\ \alpha_0 = \frac{\pi}{2},\ \dot{\alpha}_0 = 0\,, \tag{36}$$

then the debris will be removed from the collector along the corresponding halo orbit (Fig. 6b).

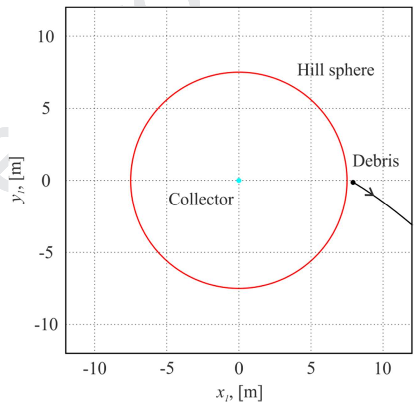
Now, let us consider the case when the debris is still outside the Hill sphere $(\rho_0 > r_H)$ and the initial conditions are determined by (36), but the engine thrust of the collector is non-zero

$$P = 0.0012\,N;\ 0.0015\,N \tag{37}$$

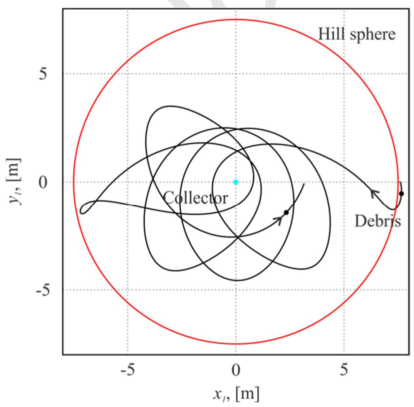
In this case, we can see a steady capture of the debris by the collector (Figs. 6c, 6d).



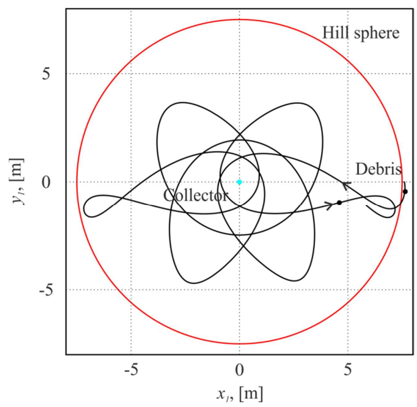
a) The thrust force $P = 0$ and the initial conditions (35).



b) The thrust force $P = 0$ and the initial conditions (36).



c) The thrust force $P = 0.0012\,N$ and the initial conditions (36).



d) The thrust force $P = 0.0015\,N$ and the initial conditions (36).

Fig. 6. The debris trajectories relative to the collector.

Fig. 6 illustrates some "ideal" pictures, when the collector is shown as a material point. In fact, with such relatively small size of the Hill sphere, it is impossible to represent the collector as the material point. We define the collector in the form of a disk with the radius r_C.

To determine the impact interaction of the small space debris and heavy collector, we introduce several additional assumptions [20]:

1. The impact does not cause any change in the position of the collector, since according to (12) mass of the small space debris is significantly less than mass of the collector.

2. The collector surface is perfectly smooth (no friction) that at the instant of contact of the mass m with the obstacle the only instantaneous force (impact force) is the normal force and the tangent force is equal to zero.

According to oblique impact model of the debris with the collector, the normal projection of velocity of the debris after impact is determined as [20]

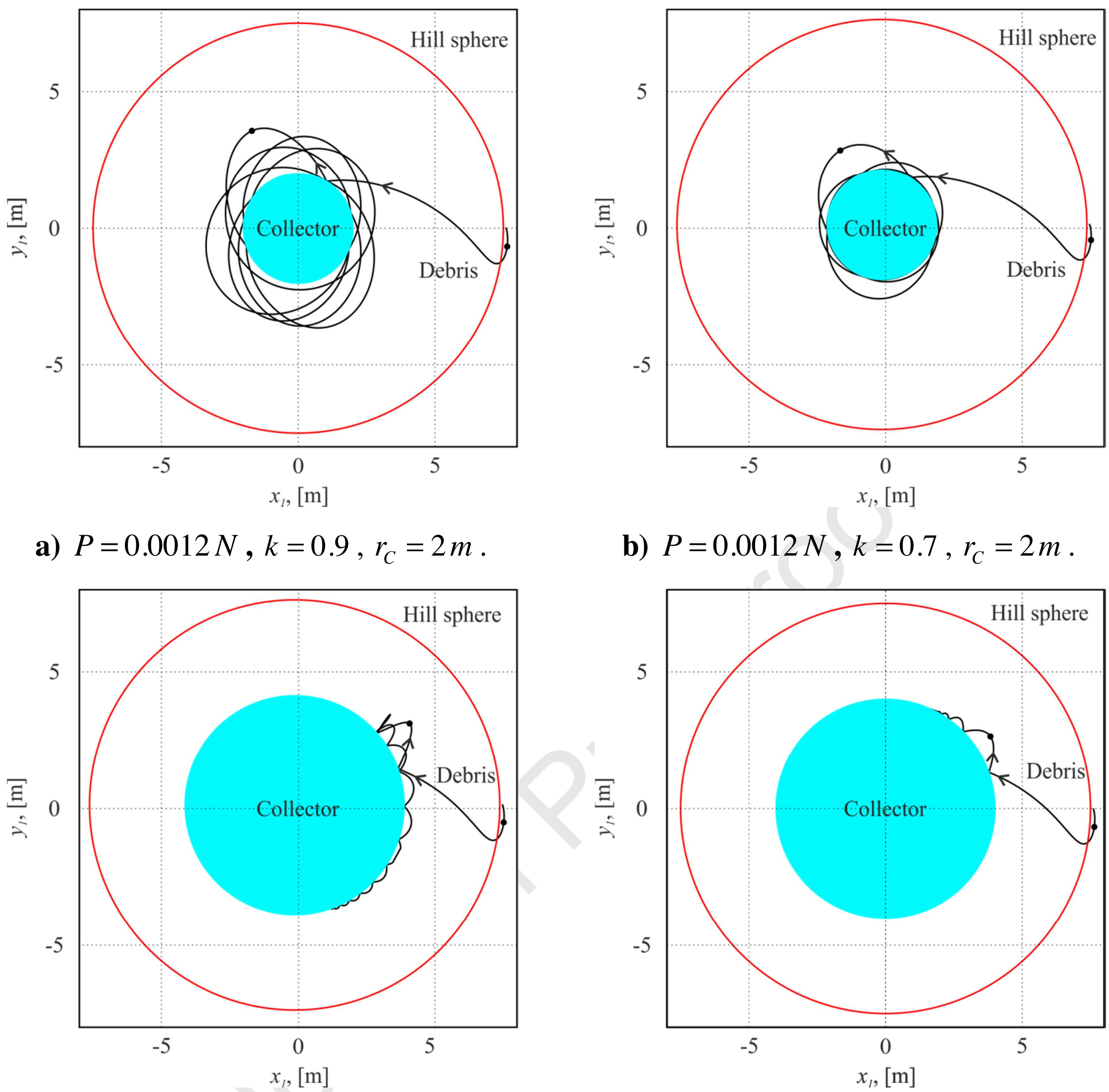
$$V_{n1} = -kV_{n0}, \tag{38}$$

where $V_{n0} = \dot{\rho}_0$, $V_{n1} = \dot{\rho}_1$ are normal projection of the velocities just before and right after impact; $k \in (0,1)$ is the coefficient of restitution at the impact (the absolutely elastic interaction $k=1$, the absolutely inelastic interaction $k=0$). Note that the tangential projections of velocity before impact and after are equal to each other [20].

Fig. 7 shows that the debris is captured at a low-thrust acceleration of the collector

$$a_1 = \frac{P}{m_1} = 1.2 \cdot 10^{-8}\, m\, s^{-2}, \tag{39}$$

and that a loss of the energy at the impact $(k<1)$ leads to the fact that the debris is located close to the collector. After numerous ricochets, the debris practically "sticks" to the collector surface and then it can be towed to the graveyard orbit.



a) $P = 0.0012\,N$, $k = 0.9$, $r_C = 2\,m$.

b) $P = 0.0012\,N$, $k = 0.7$, $r_C = 2\,m$.

c) $P = 0.0012\,N$, $k = 0.9$, $r_C = 4\,m$.

d) $P = 0.0012\,N$, $k = 0.7$, $r_C = 4\,m$.

Fig. 7. The debris trajectories relative to the collector for the initial conditions (36).

6. Discarding collected debris in the Graveyard Orbit

At the unloading phase the debris must leave the Hill sphere. The boundary value of the acceleration of the collector is denoted as a_1^*. At the towing phase it is required to satisfy the condition

$$a_1 < a_1^* \tag{40}$$

and at the unloading phase the condition is

$$a_1 > a_1^* . \tag{41}$$

If the collector does not rotate relative to the coordinate system, and if the debris object is placed at point A (Fig. 8a), then the force of inertia (from the thrust acceleration) presses the debris to the collector. In this case the debris at any acceleration will never be separated from the collector. Obviously, point C is the most convenient location of the debris in the sense of its separation due to the inertia force (Fig. 8c).

The solution of this problem can be realized by forced rotation of the collector about the longitudinal axis, which can turn any debris object to the position of point C. At point C in accordance with Eq. (20) for

$$\alpha = \pi, \dot{\alpha} = 0 \tag{42}$$

we have

$$a_1^* = \frac{P^*}{m_1} \approx \frac{G\left(m_1 + m_2\right)}{\rho^2} \tag{43}$$

where P^* is the engine thrust, sufficient for the separation of the debris from the position of the point C.

Let us consider the possibility of separating from the surface of the non-rotating collector:

$$\rho_0 = r_C = 4.0\, m,\ \dot{\rho}_0 = 0,\ \dot{\alpha}_0 = 0 , \tag{44}$$

Point A (Fig. 8a):

$$\alpha_0 = 0 . \tag{45}$$

Point B (Fig. 8b):

$$\alpha_0 = \frac{\pi}{2} . \tag{46}$$

Point C (Fig. 8c):

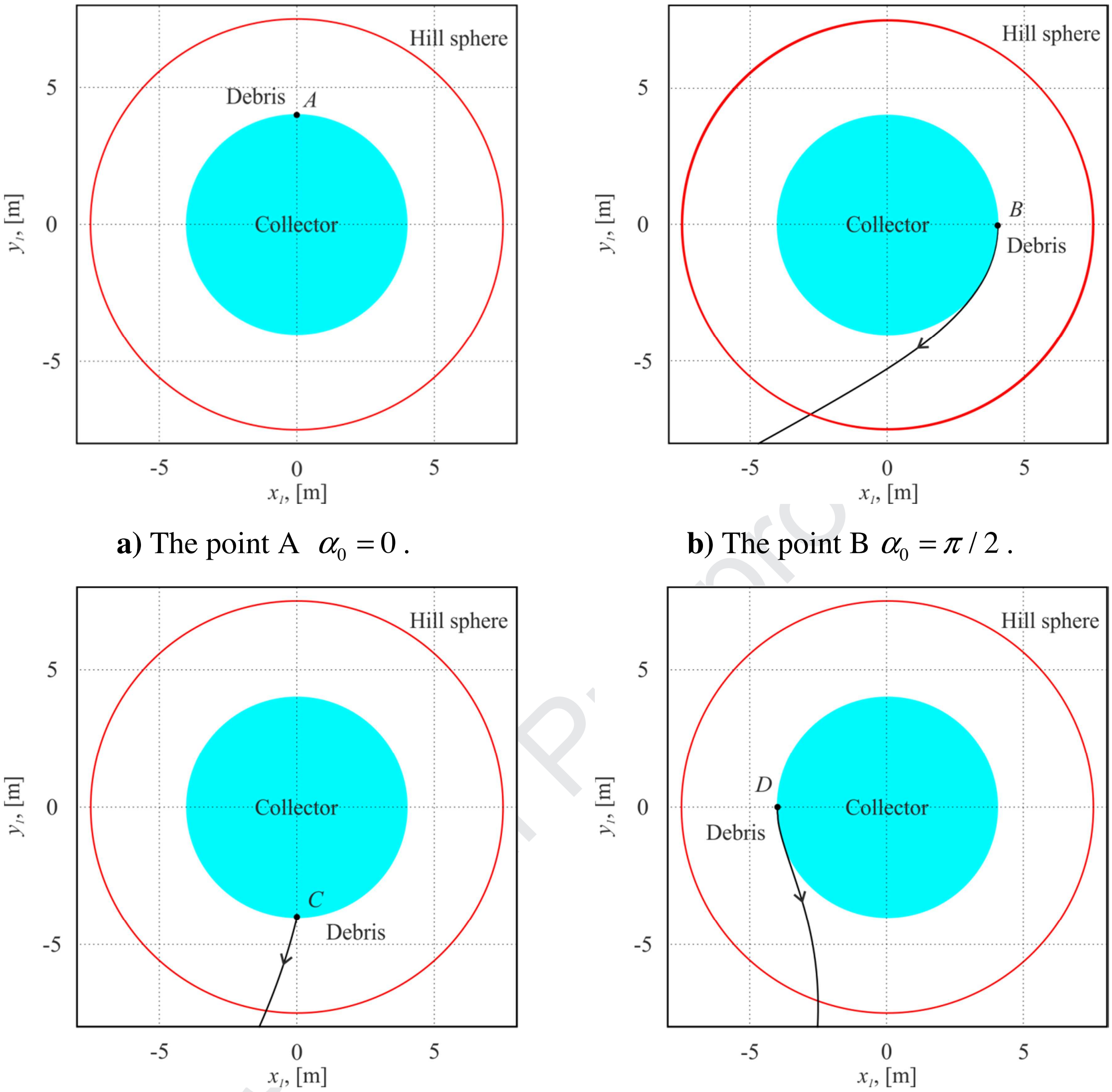
$$\alpha_0 = \pi . \tag{47}$$

Point D (Fig. 8d):

$$\alpha_0 = \frac{3}{2}\pi . \tag{48}$$

The engine thrust exceeds the limit value

$$P = 0.042\, N > P^* = 0.0417\, N . \tag{49}$$



a) The point A $\alpha_0 = 0$.

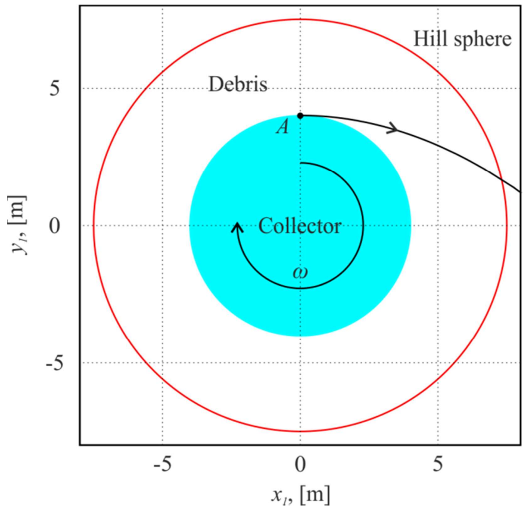
b) The point B $\alpha_0 = \pi/2$.

c) The point C $\alpha_0 = \pi$.

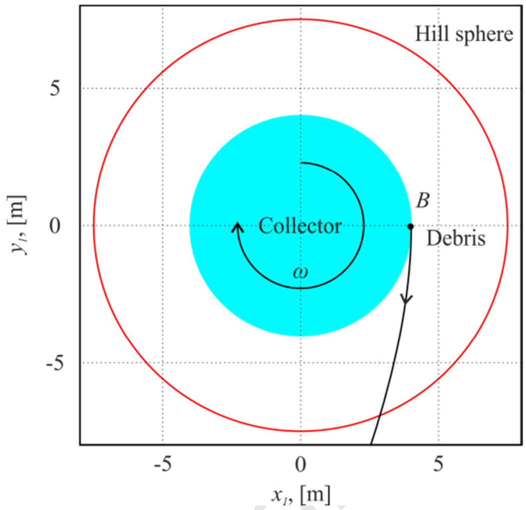
d) The point D $\alpha_0 = 3\pi/2$.

Fig. 8. The debris trajectories relative to the collector for the initial conditions (44).

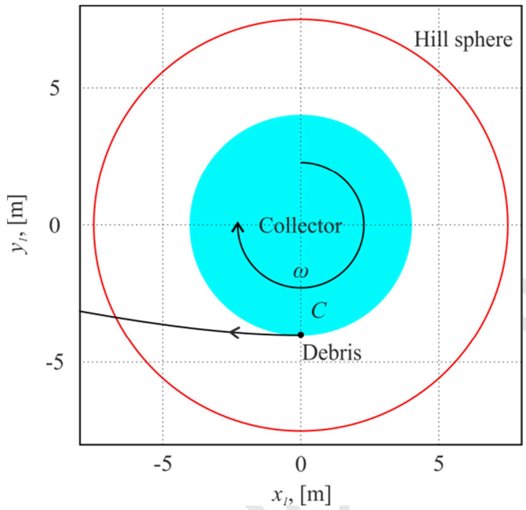
Now consider the case when the collector rotates at a low angular velocity ω, for example, equal to 0.001 rad/s (Fig. 9). Fig. 9 shows that the debris leaves the surface of the rotating collector from arbitrary point including the most unfavorable point A.



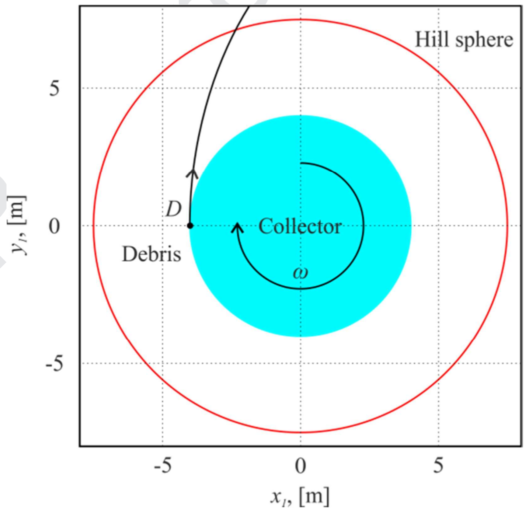
a) The point **A** $\alpha_0 = 0$, $\omega = 0.001$ rad / s .



b) The point B $\alpha_0 = \pi / 2$, $\omega = 0.001$ rad / s .



c) The point **C** $\alpha_0 = \pi$, $\omega = 0.001$ rad / s .



d) The point D $\alpha_0 = 3\pi / 2$, $\omega = 0.001$ rad / s .

Fig. 9. The debris trajectories relative to the collector at the collector rotation.

7. Requirements for the collector configuration and its attitude motion

The task of this section does not consist in the detailed description of the collector configuration, but only in the formulation of design requirements for the collector configuration and its attitude motion. Here we present the requirements that must be implemented to create the gravitational collector and why:

1. The collector should have an axisymmetric prolate cylindrical shape. This allows you to have the relatively a small diameter cylinder at the large mass.

2. The outer cylindrical surface of the collector must be sufficiently firm because it is subjected to impacts of debris, although at a very small velocity [13].

3. The collector should be constructed according to the scheme axial gyrostat [21], to implement a stable position of the collector symmetry axis perpendicular to the orbit plane. In this case at the stage of the debris separation from the collector in a graveyard orbit, the required rotation of the outer surface of the cylindrical collector can be realized through rotating a rotor [21].

8. Conclusions

The paper develops the idea of forming artificial Lagrange points and zones of attraction in the form of Hill spheres by heavy collectors in GEO to collect and remove small space debris objects as a result of a collision between satellites.

The paper does not say that the collector of the order of 100 tons will be launched in the near future. However, the following three facts can be presented, suggesting that the 100 tons collector is not such a fantasy: the mass of the ISS is 417,289 tons; the mass of all orbital objects on GEO is close to 2500 tons [1]; a space elevator has been talked about for over 70 years, but for now it remains just ideas. Moreover, a mass of the Earth’s space elevator is assumed to be more than 10,000 tons and a length of about 100,000 km. Detailed calculations have allowed to understand the fuel consumption for such mission. In addition, it is shown that the 100-ton collector transfer GEO-graveyard orbit (above GEO at 200 km) requires only 45.4 kg propellant mass.

Fears that the collector will “attract” functioning satellites may be rejected because, first, the collector has control and is able to choose targets to capture; secondly, even with such the 100-ton collector, the Hill’s sphere radius is relatively small, only 7.5 meters. Outside of this distance, the collector also affects other orbital objects, but this effect is extremely small and can be attributed to small disturbances that, along with others, affect operating satellites, and can be controlled by the control system of these satellites. On the other hand, if an asteroid delivered into GEO will be used as a collector (in the distant future), then this asteroid will certainly affect many orbital functioning and non-functioning objects. And this effect will depend on the mass of the asteroid. Then there will be a new challenge, which can only be solved jointly by all the participants of this global space mission. And finally, , one can imagine the following scenario: As a result of a collision between two satellites in GEO, a cloud of small debris has formed. In the immediate vicinity of this cloud, a single heavy collector will be assembled from several identical elements. Such a collector can be represented as a composite cylinder. Next comes the capture and delivery of this debris into a graveyard orbit. After that, the collector is dismantled into separate elements, which can be reassembled during the re-mission. In a disassembled state, the collector will not have a noticeable impact on other space objects in GEO.

The following new results were obtained:

- The mathematical models of the motion of the debris relative to the collector and the collector motion under the action of the engine thrust are proposed. The collector was considered as a cylindrical body and a small space debris object as a material point that can collide with the collector. Their collision is described by the impact theory.
- With the use of these mathematical models, the principal possibility of capturing debris in GEO, towing and unloading debris into graveyard orbit is shown. The engine thrust force is found that provide capturing and towing debris or discharging debris. In addition, the time and the propellant mass required to complete the continuous-thrust GEO–GO transfer are calculated.
- The requirements for the collector configuration and its attitude motion are formulated, which provide for the capture, towing and discharging of space debris.

Probably, the proposed space debris collection method will have some practical potential to solving the problem of cleaning the near-Earth space in future missions.

Acknowledgments

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Highlights

- Removal space debris by heavy collector from GEO is considered.
- Hill's Sphere collector is used as a capturing area of debris.
- The collector delivers debris to a graveyard orbit.
- Numerical simulations are shown, that the proposed maneuvers can be implemented.