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A splitting of collinear libration points in circular restricted three-body problem by an artificial electrostatic field

Vladimir S. Aslanov

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Abstract This paper focuses on the study of a new type of a planar, circular restricted three-body problem with an attractive artificial electrostatic field (E-field) at collinear libration points. For instance, this attractive field can be generated by an orbiting spacecraft located at the Mars-Phobos L1 libration point and an electrostatic capsule launched from Phobos. The feasibility of the proposed retrieval system is discussed from the aspect of local space weather Debye length. The attractive E-field splits the collinear libration point into two new collinear points, and the greater the E-field potential and the Debye length the greater the distance between the new libration points and the "old" original libration point. The new equilibrium positions caused by the action of the E-field have been found, and an instability of these new libration points has been proven. A new Jacobi integral in analytical form is obtained and equations of motion are derived for the restricted problem of three bodies taking into account the E-field. A numerical simulation shows the impact of the E-field potential on capsule capture in the small vicinity of the Mars-Phobos L1 libration point. This work expands the classic three-body problem filling with new content. The obtained results can be applied, for example, to study an opportunity of delivering the Phobos samples using Coulomb interaction of bodies in space.

V. S. Aslanov (✉)
Samara National Research University, 34, Moscovskoe shosse, Samara, Russia 443086
e-mail: aslanov_vs@mail.ru
URL: http://aslanov.ssau.ru/

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1 Introduction

The planar circular restricted three-body problem is a classic celestial mechanics issue. Its study made a great contribution to the theory of space dynamics and celestial mechanics. The restricted three-body problem is the basis for solving many applied tasks in astronautics, in particular, for calculating interplanetary flights and launching spacecraft and satellites. This problem and its various aspects received great attention from the scientific community. A detailed analysis of major studies on this topic can be found in the Szebehely's textbook [1]. The book discusses the regularization of the motion equations, manifold of the states of motion, equilibrium positions, motion near these positions, application of Hamiltonian dynamics methods to the restricted problem, its periodic orbits and quantitative aspects. A detailed numerical analysis of three-dimensional periodic halo orbits near collinear libration points in the restricted three-body problem was performed by Howell [2]. Analytical

Journal : Medium 11071	Dispatch : 29-1-2021	Pages : 10
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solutions for Lissajous and halo orbits near collinear points L_1, L_2 were obtained by Luo et al. in [3] using normalization method. These results were compared with the solutions obtained by Lindstedt–Poincare method. Numerical analysis of the planar circular restricted three-body problem phase space and orbits classification into three groups (bounded, escape and collisional) were performed by Zotos [4]. Halo, Lyapunov and vertical orbits for elliptic restricted three-body problem were studied based on resonant motions in the circular problem by Ferrari and Lavagna [5]. Woo and Misra [6] investigated the spacecraft motion in the vicinity of a binary asteroid system as the circular restricted case. The asteroids were considered as rigid bodies. Addition equilibrium points were found numerically for some special cases. Biggs and Negri [7] considered solar sail spacecraft controlled motion within the circular restricted three-body problem. The spacecraft moves in the gravitational field of the Earth and the Moon, taking into account the perturbation introduced by the solar pressure on the sail. Alessi and Sánchez [8] presented a semi-analytical approach, which is based on a perturbation procedure, for study the three-dimensional motion of a negligible mass body in the circular restricted three-body problem. At this point, it should be emphasized that the above references are not an exhaustive analysis of the literature on the considered topic, which is very broad and includes a huge number of works, but they give some insight into this problem. New engineering ideas require solutions of new fundamental problems. For example, in recent years, Martian sample return missions were actively discussed by scientists [9]. The missions assume that a sample capsule is launched from Mars surface using a Mars Ascent Vehicle after samples are collected by a rover [10]. As a solution to a deep-space docking challenge, the ability to use electrostatic force to capture the container was discussed in [9]. The idea of an electrostatic capture of the capsule can be used also in the case when an orbital spacecraft (orbiter) is located, in contrast to [9], at one of the collinear libration points.

If we talk about Mars, then it can be point L_3, and if about Mars's moon, Phobos, then points L_1 and L_2 are more suitable. The Phobos exploration is of independent importance. Phobos is a small, irregularly shaped moon ($\sim$ 26 $\times$ 22.8 $\times$ 18.1 km) that orbits Mars every 7 h and 39 min. The orbit is synchronous to its rotation so that its long axis is always directed toward Mars. The Mars-Phobos L_1 libration point is unusually close to Phobos' surface ($\sim$ 3.4 km), and it make the capsule delivery mission to a Phobos Sample Return orbiter technically feasible if the orbiter hovered in the Mars-Phobos L_1 point. Several theoretical questions have to be answered in order to investigate a possibility of using the attractive artificial E-field at the collinear libration points L_1, L_2 and L_3, for example, the Phobos sample capsule delivery mission.

The answers to these questions are covered in this article. This study focuses on equations and analytical formulas describing the new planar circular restricted three-body problem with the additional E-field. The goal is to understand the body's behavior in the E-field and the two gravitational fields caused by the main bodies near the collinear libration points. Note that when considering the E-field, the Debye length λ_D must be taken into account. The Debye length is an important parameter because the E-field rapidly decreases beyond this length by the Debye shielding effect. The motion equations are written in a rotating Cartesian coordinate system, which is converted to a canonical dimensionless form. We show the splitting of the "old" collinear liberation points L_1, L_2, L_3, and find new paired equilibrium positions (L_{1+}, L_{1-}; L_{2+}, L_{2-}; L_{3+}, L_{3-}) in the vicinity of the "old" points, and also prove the instability of these new equilibrium positions. Next, a new Jacobi integral is obtained in analytical form for the circular restricted three-body problem taking into account the E-field. And finally, using numerical modeling, we show the effect of the E-field potential on the capture of the electrostatic capsule (E-capsule) in the small vicinity of the Mars-Phobos L_1 libration point.

2 Equations of motion and Jacobi integral

In this section, we derive the planar motion equations of a body in two gravitational and one attractive E-field. The center of E-field is located at a single collinear libration point. These points are called the "old" collinear libration points because, as can be assumed (this will be proved below), due to the action of the E-field, the "old" unstable point splits into two new unstable equilibrium points. The action of the E-fields is limited to the Debye length within tens of meters. In addition, the motion of a body is studied

Journal : Medium 11071 | Dispatch : 29-1-2021 | Pages : 10
Article No. : 6226 | LE | TYPESET
MS Code : NODY-D-20-02999R1 | CP | DISK

only in a small vicinity of "old" points comparable to Debye length, beyond which the problem degenerates into the classic three-body problem. Therefore, we talk about two gravity fields and one attractive E-field, and the motion equations are written for the vicinity of each point:L_1, L_2, L_3. The equations of motion are derived using classical terminology of the three-body problem presented in book by Schaub and Junkins [11]. This will facilitate understanding of the presented material. Below this section is organized as follows. Firstly, the conventional assumptions for the circular restricted three-body problem are described. Next, the expression for Coulomb force acting on the charged body in the E-field is given. Then, the equations of the body motion are derived in canonical form. Finally, the non-dimensional potential function and the Jacobi integral are found.

The considered mechanical system consists of three bodies M_1, M_2 and M(Fig. 1). It is assumed that the mass of the body M is many times less than the mass of the bodies M_1 and M_2. Therefore, the body M has negligible effect on the other bodies. It is also supposed that the bodies M_1 and M_2 move in circular orbits around their mutual center of mass. Before proceeding with the construction of the relative motion equations, the electrostatic force is introduced. The orbiter is assumed to be located at one of the collinear libration points of M_1- M_2 bodies system and to have an electrostatic charge. The mass of the orbiter is not taken into account. The charged electrostatic capsule (E-capsule) is affected by an electrostatic force when moving in the vicinity of the orbiter. The capsule performance is dependent on this force. It is assumed that the capsule and the orbiter are perfectly

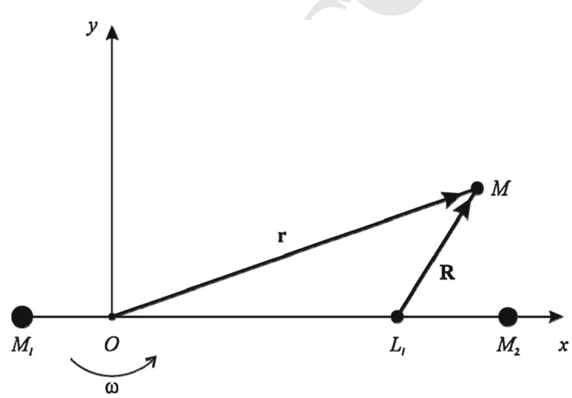

Fig.1 The Local-Vertical-Local-Horizontal frame *Oxy*

conducting spheres. The electrostatic force between the capsule and orbiter is defined by the equation

$$F = k_C \frac{q_O q}{R^2} \tag{1}$$

where $k_C = 8.99 \cdot 10^9 \mathrm{Nm}^2/C^2$ is the Coulomb constant, q_O is the charge on the orbiter, q is the charge on the capsule, R is the distance between the capsule and orbiter. Note that this force may be either attractive or repulsive depending on the polarity of the potentials of the E-capsule and orbiter. Consider only the attracting configuration $(q_O q < 0)$ in this paper.

Let us first introduce into consideration three vectors in the Local-Vertical-Local-Horizontal frame *Oxy*: the position vector of the body M (Fig. 1)

$$\mathbf{r} = (r_x, r_y) \tag{2}$$

the vector $\mathbf{a} = (a, 0)$ determines the position of the collinear libration points L_1, L_2 and L_3 in the frame *Oxy*

$$a = (a_1, a_2, a_3), \quad a_i = \left|\overline{OL_i}\right|, \ i = 1, 2, 3 \tag{3}$$

and, the position vector of the body M with respect to the collinear libration point (Fig. 1)

$$\mathbf{R} = \mathbf{r} - \mathbf{a} = (X = r_x - a, Y = r_y) \tag{4}$$

Then, the vector of the Coulomb force (1) is written as

$$\mathbf{F} = kF\frac{\mathbf{R}}{R} = k\frac{k_C q_O q}{R^3}\mathbf{R}, \quad (q_O q < 0) \tag{5}$$

Note that the Coulomb force affects the body M, if this body is within the Debye length of a collinear libration point, i.e., within 10 s meters

$$k = \begin{cases} 1, & \sqrt{(r_x - a)^2 + r_y^2} \le \lambda_D \\ 0, & \sqrt{(r_x - a)^2 + r_y^2} > \lambda_D \end{cases} \tag{6}$$

Now, taking into account the Coulomb force (5), the equations of the E-capsule planar motion can be written the frame *Oxy*[11]

$$\ddot{r}_x = \frac{\partial W}{\partial r_x} + \omega^2 r_x + 2n\dot{r}_y + k\frac{k_C q_O q}{m}\frac{(r_x - a)}{R^3} \tag{7}$$

$$\ddot{r}_y = \frac{\partial W}{\partial r_y} + \omega^2 r_y - 2n\dot{r}_x + k\frac{k_C q_O q}{m}\frac{r_y}{R^3} \tag{8}$$

Journal : Medium 11071 | Dispatch : 29-1-2021 | Pages : 10
Article No. : 6226 | ☐ LE | ☐ TYPESET
MS Code : NODY-D-20-02999R1 | ☑ CP | ☑ DISK

$$W(r_x, r_y) = G\left(\frac{m_1}{\sqrt{(r_x + d\mu)^2 + r_y^2}} + \frac{m_2}{\sqrt{(r_x - d(1-\mu))^2 + r_y^2}}\right) \tag{9}$$

where $\mu = \frac{m_2}{m_1+m_2}$; ω is a constant angular velocity magnitude of the M_1–M_2 system; d is the distance between M_1 and M_2; m, m_1 and m_2 are masses of the bodies M, M_1 and M_2, respectively.

The motion equations (7) and (8) can be written in a convenient non-dimensional form. To do so, we introduce the non-dimensional time variable t as

$$\tau = \omega t \tag{10}$$

Time derivatives with respect to this new time variable are denoted as

$$(.)' = \frac{d}{d\tau}(.) \tag{11}$$

Any scalar distances are non-dimensionalized by dividing them with the constant relative between M_1 and M_2 as

$$x = \frac{r_x}{d},\ y = \frac{r_y}{d} \tag{12}$$

Using the variables substitutions (10) and (12), we now are able to rewrite Eqs. (7) and (8) into the following non-dimensional form:

$$x'' - 2y' = \frac{\partial U}{\partial x} \tag{13}$$

$$y'' + 2x' = \frac{\partial U}{\partial y} \tag{14}$$

where $\alpha = \frac{a}{d} < 1$. The corresponding non-dimensional potential function $U(x, y)$ is given by the expression

$$U(x,y) = \frac{1}{2}(x^2+y^2) + \frac{1-\mu}{\sqrt{(\mu+x)^2+y^2}} + \frac{\mu}{\sqrt{(-1+\mu+x)^2+y^2}} - k\frac{\Phi}{\sqrt{(x-\alpha)^2+y^2}} \tag{15}$$

where

$$\Phi = \frac{k_C q_O q}{m d^3 \omega^2} < 0 \tag{16}$$

Due to the definition of the mass ratio μ the non-dimensional coordinates of M_1 and M_2 are written as.

$$x_1 = -\mu, \quad x_2 = 1 - \mu \tag{17}$$

This expression is simplified by entering a non-dimensional relative distance ρ_i is defined as

$$\rho_i = \sqrt{(x-x_i)^2 + y^2}, \quad \rho = \sqrt{(x-\alpha)^2 + y^2} \tag{18}$$

Then, the corresponding non-dimensional potential function (15) is given by the expression

$$U(x,y) = \frac{1}{2}(x^2+y^2) + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} - k\frac{\Phi}{R} \tag{19}$$

Following similar steps as were done with the classic restricted three-body problem [11], with the E-field the non-dimensional Jacobi integral takes on the form

$$J(x,y) = (x^2+y^2) + \frac{2(1-\mu)}{\sqrt{(\mu+x)^2+y^2}} + \frac{2\mu}{\sqrt{(-1+\mu+x)^2+y^2}} - 2k\frac{\Phi}{\sqrt{(x-\alpha)^2+y^2}} - (x'^2+y'^2) \tag{20}$$

For the dimensional equations of motion (13) and (14) the non-dimensional Jacobi integral is written as

$$J(x,y) = (x^2+y^2) + \frac{2(1-\mu)}{\rho_1} + \frac{2\mu}{\rho_2} - 2k\frac{\Phi}{\rho} - (x'^2 + y'^2) \tag{21}$$

3 New Lagrange collinear libration points

Setting the relative velocities and accelerations in Eqs. (13) and (14) equal to zero, we find conditions that are satisfied by the stationary points of the circular restricted three-body problem with the E-field. Obviously, the new stationary points can be located only within the Debye length of the "old" collinear libration points L_i $(i = 1, 2, 3)$, i.e., within tens of meters. Outside this boundary, this task degenerates into the classic circular restricted three-body problem when, $k = 0$ according to Eqs. (6), (13) and (14).

Journal : Medium 11071 | Dispatch : 29-1-2021 | Pages : 10
Article No. : 6226 | LE | TYPESET
MS Code : NODY-D-20-02999R1 | CP | DISK

Therefore, we will look for new libration points only inside the Debye sphere is defined as

$$\sqrt{(r_x - a)^2 + r_y^2} \le \lambda_D \tag{22}$$

or, taking into account (12)

$$\sqrt{(x - \alpha)^2 + y^2} \le \frac{\lambda_D}{d} = l_D \tag{23}$$

To solve for the scalar coordinate x for the collinear libration points, Eq. (13) is set equal to zero for $y' = y = 0$, simplify to following [11]

$$0 = x - \frac{\mu(-1 + \mu + x)}{(-1 + \mu + x)^3} - \frac{(1 - \mu)(\mu + x)}{(\mu + x)^3} + k\frac{\Phi(x - \alpha)}{(x - \alpha)^3} \tag{24}$$

Let's rewrite Eq. (24) with regard to (17) as

$$0 = x - \frac{\mu(x - x_2)}{(x - x_2)^3} - \frac{(1 - \mu)(x - x_1)}{(x - x_1)^3} + k\frac{\Phi(x - \alpha)}{(x - \alpha)^3} \tag{25}$$

Using the fact that the new libration points generated by the E-field must be located within the Debye sphere (23), we begin with the vicinity of the point L_1 and find from Eq. (25) implicit conditions for the L_{1-} and L_{1+} position coordinates in terms of the mass ratio μ:

$$L_{1-}:\ 0 = x - \frac{1 - \mu}{(\mu + x)^2} + \frac{\mu}{(x - 1 + \mu)^2} - \frac{\Phi}{(x - \alpha)^2} \tag{26}$$

$$L_{1+}: 0 = x - \frac{1 - \mu}{(\mu + x)^2} + \frac{\mu}{(x - 1 + \mu)^2} + \frac{\Phi}{(x - \alpha)^2} \tag{27}$$

The last terms differentiate these equations from Eq. (10.93) in [11] and show the splitting of the point L_1 into two new unstable collinear points L_{1-} and L_{1+}. Now consistently, in a similar way, we write down the implicit conditions for the collinear libration points in the vicinity of the "old" libration points L_2 and L_3:

$$L_{2-}: 0 = x - \frac{1 - \mu}{(\mu + x)^2} - \frac{\mu}{(x - 1 + \mu)^2} - \frac{\Phi}{(x - \alpha)^2} \tag{28}$$

$$L_{2+}: 0 = x - \frac{1 - \mu}{(\mu + x)^2} - \frac{\mu}{(x - 1 + \mu)^2} + \frac{\Phi}{(x - \alpha)^2} \tag{29}$$

and

$$L_{3-}: 0 = x + \frac{1 - \mu}{(\mu + x)^2} + \frac{\mu}{(x - 1 + \mu)^2} + \frac{\Phi}{(x - \alpha)^2} \tag{30}$$

$$L_{3+}: 0 = x + \frac{1 - \mu}{(\mu + x)^2} + \frac{\mu}{(x - 1 + \mu)^2} - \frac{\Phi}{(x - \alpha)^2} \tag{31}$$

The order of the location of the new libration points caused by the E-field is given in Fig. 2.

4 Libration points stability

To study the stability of the new equilibrium positions (L_{1+}, L_{1-} L_{2+}, L_{2-} L_{3+}, L_{3-}) caused by the E-field, we use a standard linearization procedure [12]. Firstly, the equations of relative motion are linearized in the small vicinity of the new equilibrium position. Then, the eigenvalues of the linearized plant matrix are determined. The conclusion about the equilibrium position stability is made on the basis of the real part of these eigenvalues. Let denote the position of the Lagrange point as (x_0, y_0). The body M is located at the point with coordinates $(x_0 + \xi, y_0 + \eta)$. Calculating the body velocity components (ξ', η'), substituting these quantities into Eqs. (13), (14) and expanding result in a Taylor series gives

$$\xi'' - 2\eta' = \xi\frac{\partial^2 U}{\partial x^2}\bigg|_0 + \eta\frac{\partial^2 U}{\partial x \partial y}\bigg|_0 + \cdots \tag{32}$$

$$\eta'' + 2\xi' = \xi\frac{\partial^2 U}{\partial x \partial y}\bigg|_0 + \eta\frac{\partial^2 U}{\partial y^2}\bigg|_0 + \cdots \tag{33}$$

where the suffix zero means that after the partial If the displacements ξ and η are small, we may neglect terms involving squares, products and higher-degree terms in ξ and η, and so the equations become

$$\xi'' - 2\eta' = \xi U_{xx} + \eta U_{xy} \tag{34}$$

$$\eta'' + 2\xi' = \xi U_{yx} + \eta U_{yy} \tag{35}$$

where

Journal : Medium 11071 Dispatch : 29-1-2021 Pages : 10
Article No. : 6226 LE TYPESET
MS Code : NODY-D-20-02999R1 CP DISK

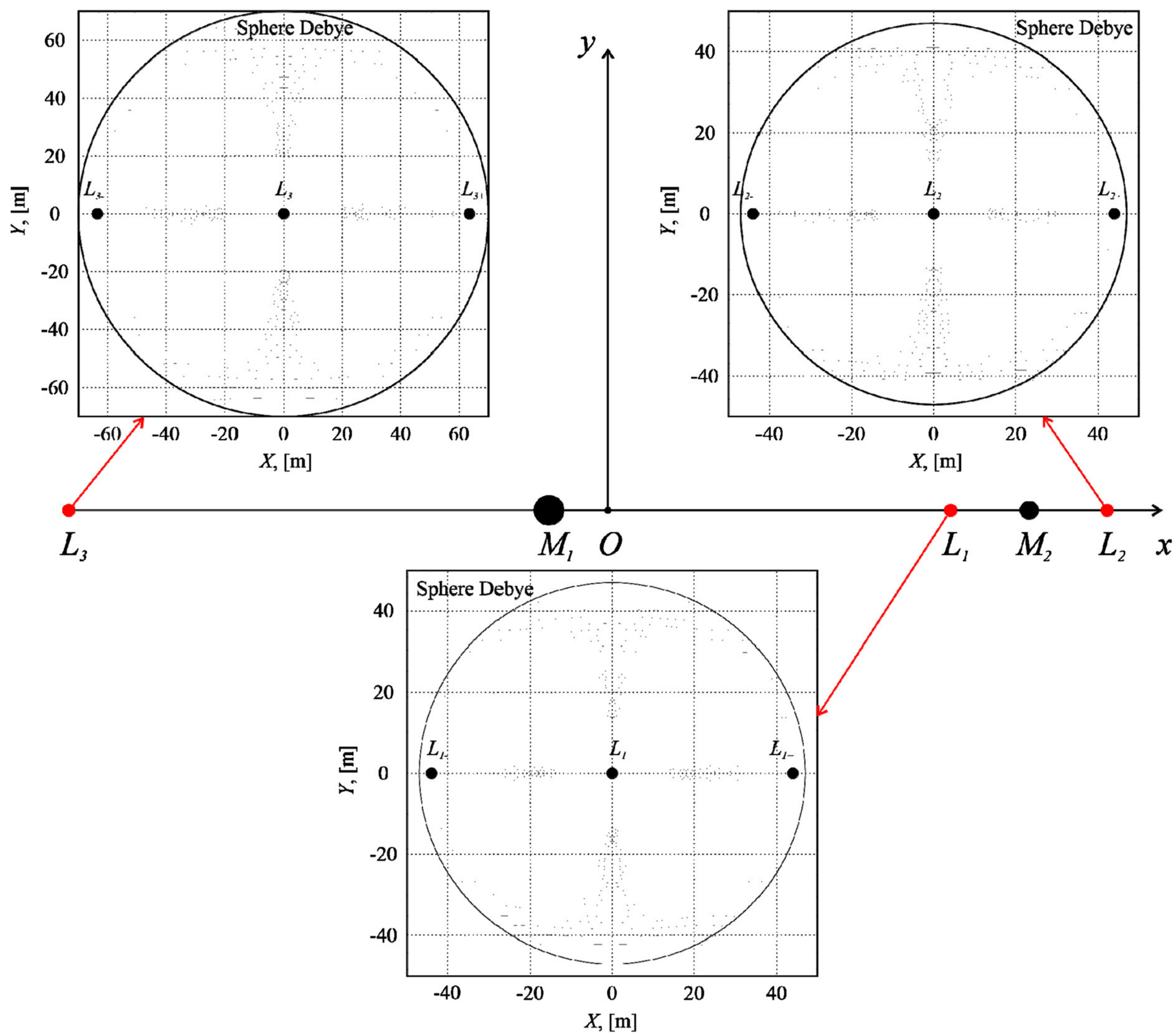

Fig. 2 Splitting the collinear libration points, when $\Phi = \frac{k_C q_O q}{m d^3 \omega^2} = -9 \cdot 10^{-16}$

$$U_{xx} = \left.\frac{\partial^2 U}{\partial x^2}\right|_0, \quad U_{yy} = \left.\frac{\partial^2 U}{\partial y^2}\right|_0, \quad U_{xy} = U_{yx} = \left.\frac{\partial^2 U}{\partial x \partial y}\right|_0 = \left.\frac{\partial^2 U}{\partial y \partial x}\right|_0 \tag{36}$$

and the U are constant since they are evaluated at the Lagrange point. These are linear differential equations with constant coefficients, the general solution of which may be written as

$$\xi = \sum_{i=1}^{4} \alpha_i \exp(\lambda_i t), \quad \eta = \sum_{i=1}^{4} \beta_i \exp(\lambda_i t) \tag{37}$$

where α_i are integration constants, the constants β_i dependent upon α_i and the constants appearing in the differential equations. The λ_i are the roots of the characteristic determinant of Eqs. (34) and (35) set equal to zero and rewritten as

$$\begin{vmatrix} \lambda^2 - U_{xx} & -2\lambda - U_{xy} \\ 2\lambda - U_{xy} & \lambda^2 - U_{yy} \end{vmatrix} = 0 \tag{38}$$

or

$$\lambda^4 + (4 - U_{xx} - U_{yy})\lambda^2 + U_{xx}U_{yy} - U_{xy}^2 = 0 \tag{39}$$

The solution is stable when all λ_i obtained from Eq. (39) are pure imaginary numbers. Since, along with any root λ, the biquadratic characteristic equation also has a root $-\lambda$, then the solution is unstable when

Journal : Medium 11071 | Dispatch : 29-1-2021 | Pages : 10
Article No. : 6226 | ☐ LE | ☐ TYPESET
MS Code : NODY-D-20-02999R1 | ☑ CP | ☑ DISK

any of the λ_i are real or complex numbers with a nonzero real part. Now, defining the following quantities A, B, C, D and E as

$$A = \frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3} \tag{40}$$

$$B = 3\left(\frac{1-\mu}{\rho_1^5} + \frac{\mu}{\rho_2^5}\right) \tag{41}$$

$$C = 3\left(\frac{1-\mu}{\rho_1^5}(x_0+\mu) + \frac{\mu}{\rho_2^5}(x_0-(1-\mu))\right) \tag{42}$$

$$D = k\frac{\Phi}{|\rho|^3} \tag{43}$$

and

$$E = k\frac{\Phi}{|\rho|^5} \tag{44}$$

We find that

$$U_{xx} = 1 - A + 3(1-\mu)\frac{(x_0+\mu)^2}{\rho_1^5} + 3\mu\frac{(x_0-(1-\mu))^2}{\rho_2^5} + D - 3E(\alpha - x_0)^2 \tag{45}$$

$$U_{yy} = 1 - A + By_0^2 + D - 3Ey_0^2 \tag{46}$$

$$U_{xy} = Cy_0 + 3E(\alpha - x_0)y_0 \tag{47}$$

In the straight line solution, $y_0 = 0$, so that

$$\rho_i^2 = (x_0 - x_i)^2, \quad \rho^2 = (x_0 - \alpha)^2 \tag{48}$$

Hence

$$U_{xx} = 1 + 2A - 2D, \quad U_{yy} = 1 - A + D, \quad U_{xy} = 0 \tag{49}$$

Applying the values from (49) in Eq. (39) we obtain

$$\lambda^4 + (2 - A')\lambda^2 + (1 + A' - 2A'^2) = 0 \tag{50}$$

where

$$A' = A - D = \frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3} - k\frac{\Phi}{\rho^3} \tag{51}$$

It can be shown that

$$1 + A' - 2A'^2 < 0 \tag{52}$$

for values of μ up to its limit of $\frac{1}{2}$. Hence, the four roots of equation (50) consist of two real roots, numerically equal but opposite in sign, and two conjugate pure imaginary roots. Hence the solution is unstable. Obviously, condition (52) with quadratic dependence on the left side is satisfied for any A' greater than unity

$$A' = A - D > 1 \tag{53}$$

Let us demonstrate that all the new equilibrium positions $(L_{1+}, L_{1-}\ L_{2+}, L_{2-}\ L_{3+}, L_{3-})$ satisfy this condition. Consider the first two of these points L_{1+}, L_{1-} located within the Debye sphere with respect to the point L_1. Since these points are located between M_1 and M_2, then $\rho_1, \rho_2 < 1$ and therefore

$$A = \frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3} > 1 - \mu + \mu = 1 \tag{54}$$

Now consider the parameter D, which in consequence of (43) and (1) within the Debye sphere is always negative:

$$D = k\frac{\Phi}{|\rho|^3} < 0 \tag{55}$$

Thus, the condition (53) is satisfied for the equilibrium positions L_{1+}, L_{1-}, and hence the condition (52) is also satisfied, hence the equilibrium positions L_{1+}, L_{1-} are unstable.

We now investigate the stability at the other collinear equilibrium positions L_{2+}, L_{2-}, L_{3+} and L_{3-}, found from the four equations (28)–(31). Below we prove that at these points $A' > 1$ as well. For this purpose equation (25) can be rewritten as

$$0 = x\left(1 - \frac{1-\mu}{\rho_1^3} - \frac{\mu}{\rho_2^3}\right) + \frac{(1-\mu)}{\rho_1^3}x_1 + \frac{\mu}{\rho_2^3}x_2 + k\frac{\Phi(x-\alpha)}{(x-\alpha)^3} \tag{56}$$

Considering that $x_1 = -\mu$, $x_2 = 1 - \mu$, get

$$x\left(\frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3} - 1\right) = \mu(1-\mu)\left(\frac{1}{\rho_2^3} - \frac{1}{\rho_1^3}\right) + k\frac{\Phi(x-\alpha)}{(x-\alpha)^3} \tag{57}$$

comparing this expression with Eqs. (40), (43) and (53) write it as

Journal : Medium 11071 | Dispatch : 29-1-2021 | Pages : 10
Article No. : 6226 | ☐ LE | ☐ TYPESET
MS Code : NODY-D-20-02999R1 | ☑ CP | ☑ DISK

$$
\begin{aligned}
A' - 1 &= \frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3} - 1 - k\frac{\Phi}{|\rho|^3} \\
&= \frac{\mu(1-\mu)}{x}\left(\frac{1}{\rho_2^3} - \frac{1}{\rho_1^3}\right) - k\frac{\Phi}{|\rho|^3} + k\frac{\Phi(x-\alpha)}{x(x-\alpha)^3}
\end{aligned}
\tag{58}
$$

Note that at the points L_{2+} and L_{2-}, the conditions $x > 0$ and $\rho_2 < \rho_1$ are satisfied, and at the points L_{3+} and L_{3-}, the conditions $x < 0$ and $\rho_2 > \rho_1$. Besides

$$
-k\frac{\Phi}{|\rho|^3} + k\frac{\Phi(x-\alpha)}{x(x-\alpha)^3} > 0 \tag{59}
$$

since $\Phi < 0$ and x always many orders of magnitude greater than ρ, which does not exceed the relative Debye length (23). Therefore, in these considered cases we get

$$
A' > 1 \tag{60}
$$

It follows that condition (52) is also satisfied. Thus, we have proved that the all new collinear equilibrium points (L_{1+}, L_{1-} L_{2+}, L_{2-} L_{3+}, L_{3-}) are unstable.

5 Numerical simulation

This section shows the effect of the electrostatic charge level on the capture of the E-capsule in the vicinity of the Mars-Fobos libration point L_1, in which the orbiter together with the E-capsule generates the attracting E-field. Note that the Debye length near the L_1 point is unknown; approximate Debye length values are given for the Stickney crater, located on Phobos' surface directly under the L_1 point. Depending on Mars local time, this parameter ranges from 13 to 47 m [14]. In the absence of accurate data, all calculations are performed for $\lambda_D = 45m$. Since we study the motion of the capsule relative to the orbiter, which is in the L_1 point, it makes sense to pass from the frame Oxy to the frame L_1XY (Fig. 3) by changing the variable

$$
x = X + a_1\ ,\ y = Y \tag{61}
$$

where a_1 is abscissa of the Mars-Phobos L1 point

Then taking into account (61), the motion equations (7) and (8) can be rewritten as

$$
\ddot{X} = \frac{\partial W_E}{\partial X} + \omega^2(X + a_1) + 2\omega\dot{Y} \tag{62}
$$

$$
\ddot{Y} = \frac{\partial W_E}{\partial Y} + \omega^2 Y - 2\omega\dot{X} \tag{63}
$$

where the effective potential is written as

$$
\begin{aligned}
W_E(X,Y) = G\Bigg(&\frac{m_1}{\sqrt{(X + a_1 + d\mu)^2 + Y^2}} \\
&+ \frac{m_2}{\sqrt{(X + a_1 - d(1-\mu))^2 + Y^2}}\Bigg) \\
&- k\frac{P}{m\sqrt{X^2+Y^2}}
\end{aligned}
\tag{64}
$$

where $P = k_C q_O q < 0$ is the charge level.

All trajectories of the capsule $m = 10\,kg$ begin with the same initial conditions in the vicinity of point L_1 from the side of Phobos in the frame L_1XY

$$
\begin{aligned}
&X_0 = 81.533\,m, \quad Y_0 = 10.829\,m, \\
&\dot{X}_0 = -0.043\,m/s, \quad \dot{Y}_0 = -0.017\,m/s
\end{aligned}
\tag{65}
$$

Figure 3 demonstrates the E-capsule trajectories for different charge levels:

$$
P = k_C q_O q = 0,\ -0.28,\ -0.32,\ -0.40\ [\mathrm{N \cdot m^2}]
$$

In the first case the E-field is absent, therefore, the capsule does not reach the vicinity of the point L_1, the same thing we can see when the charge level is not high enough ($P = -0.28\,\mathrm{N \cdot m^2}$). If the charge level is equal $P = -0.32\,\mathrm{N \cdot m^2}$ and more ($P = -0.40\,\mathrm{N \cdot m^2}$), there are several turns of the capsule around the point L_1, and the greater the charge level, the more turns the E-capsule performs around the point L_1.

6 Conclusion

This paper shows within the framework of the planar circular restricted three-body problem that the artificial attractive E-field at one of the collinear libration points causes the splitting of this point into two other unstable collinear libration points located within the Debye length. This fact was proved analytically, using

Journal : Medium 11071 | Dispatch : 29-1-2021 | Pages : 10
Article No. : 6226 | ☐ LE | ☐ TYPESET
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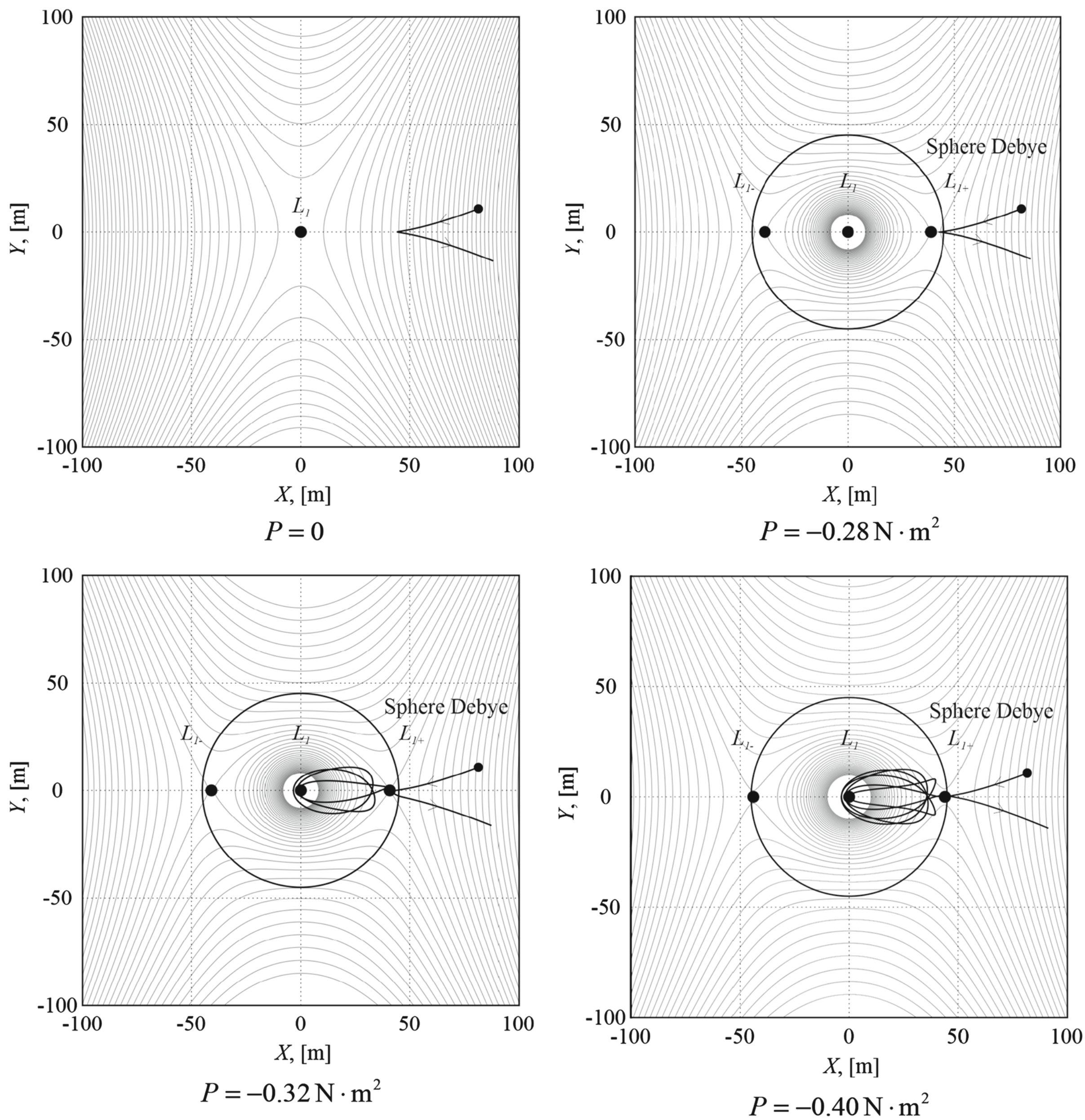

Fig. 3 The capsule trajectories and Contour plot of the effective potential $W_E(X, Y)$ in the frame L_1XY

the corresponding analytic equations, the new Jacobi integral, and confirmed by the numerical simulation. We have demonstrated the feasibility of the attracting mission the E-capsule in a small vicinity of the Mars-Fobos libration point L_1.

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Compliance with ethical standards

Conflict of interest The author declares that he has no conflict of interest.

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Journal : Medium 11071 | Dispatch : 29-1-2021 | Pages : 10
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