Advanced Mission Scenarios of the Three-Body Problem

VLADIMIR S. ASLANOV

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Advanced Mission Scenarios of the Three-Body Problem introduces one of the classical problems of celestial mechanics through the prism of the problems and challenges of modern astrodynamics. It examines current issues in modern astronautics related to the study and colonization of Mars, exploration of the Moon, space debris, and contactless space transportation technologies.

Describing the dynamics of space tether systems and space elevators in the gravitational field created by two massive bodies, the book presents an original technology for contactless gravitational transportation of space debris using a heavy gravitational collector. It provides practical recommendations for the implementation of new pioneering missions related to tether transportation in the vicinity of libration points, the construction of space elevators on planetary moons, and contactless electrostatic and gravitational transportation, including in the context of the space debris problem.

This book is intended for space systems and spacecraft engineering industry experts and researchers studying space transportation and modern challenges in astronautics.

Vladimir S. Aslanov graduated from the Kuibyshev Aviation Institute (KuAI), Kuibyshev, USSR in 1972 as a specialty Aircraft Engineer, and he later received his PhD from KuAI in 1977. Between 1978 and 1982 Aslanov worked as an assistant professor in the KuAI. From 1989 to 2023, Prof. Aslanov was the head of the Department of Theoretical Mechanics of Samara State Aerospace University, Samara, Russia. Since 2023, Prof. Aslanov has been a professor of the Department of Theoretical Mechanics of Samara State Aerospace University, Samara, Russia; and the professor of the Department of Mechatronics and theoretical mechanics of Moscow Aviation Institute (National Research University), Moscow, Russia. Prof. Aslanov's scientific interests are classical mechanics, nonlinear oscillations and chaotic dynamics, mechanics of space flight, dynamics of gyrostats, dynamics of tethered satellite systems, and spacecraft stability. He has published more than 130 papers in international journals and for conference proceedings.

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To my wife, Lyudmila

Contents

About the Author

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Prof. Aslanov's scientific interests are classical mechanics, nonlinear oscillations and chaotic dynamics, mechanics of space flight, dynamics of gyrostats, dynamics of tethered satellite systems and spacecraft stability. Prof. Aslanov has published more than 130 papers in international journals and for conference proceedings. The list of his most significant papers can be found on the website http://aslanov.ssau.ru. The results of the professor's scientific work were included in three monographs:

- V. S. Aslanov and A. S. Ledkov, Attitude dynamics and control of space debris during ion beam transportation, 2022.
- V. S. Aslanov, Rigid Body Dynamics for Space Applications, 2017.
- V. S. Aslanov and A. S. Ledkov, Dynamics of the Tethered Satellite Systems, 2012.

In present, Professor Aslanov is one of the best authors in SciVal topics: "Ion Beams; Imidazole; Space Debris; (T.58900)" (2nd place) and "Rigid Structure; Equation of Motion; Spacecraft" (3rd place). Besides his scientific activity Professor Aslanov is actively engaged in educational work. He conducted lecture courses "Analytical mechanics," "Modern problems of mechanics," and "Dynamics of the space vehicle in atmosphere." Under the guidance of Prof. Aslanov, nine of his students received a PhD degree. The high level of teaching has been confirmed by awards: the Academician S. P. Korolev Medal "For services to state cosmonautics" (Federation of Cosmonautics of Russia, Moscow, 2007), the award of the governor of the Samara region (Samara, 2002), and the Mayoral award of Samara (Samara, 1997).

Introduction

The restricted three-body problem is one of the classical problems of celestial mechanics, which was formulated by Euler in 1767. The problem involves the motion of three bodies affected by gravitational interactions among them. One of these bodies has such a small mass that it does not influence the motion of the other two. Despite the straightforward nature of the formulation, a general solution to the problem has yet to be found. The development of astronautics has made it possible to see new aspects of this problem and formulate novel scientific directions. This book is based on original results obtained by the author since 2019 and published in leading scientific aerospace and mechanical engineering journals. The first chapter gently and carefully introduces the reader to the classic restricted three-body problem theory, preparing the reader for the following chapters and lowering the educational barrier to understanding subsequent material. For a long time, the restricted three-body problem has been used to design spacecraft trajectories. The goal of this book is to present to a wide audience the new cases of the restricted three-body problem and projects for their practical implementation. The approaches and missions' schemes described in the book are based on the current level of technology and can be implemented in the near future.

The main part of the book contains new problems that unexpectedly expand the classical three-body problem, emphasizing its inexhaustibility. In addition to scientific interest, these tasks have important applied significance and were formulated in response to the challenges of modern astronautics associated with the exploration of planets and satellites, the creation of effective contactless transport systems and spacecraft for space debris removal. The reader will become acquainted with such interesting topics as the use of tether systems near libration points, space elevators deployed from the surface of the moon, the splitting of libration points in the result of the action of electrostatic forces, and contactless removal of space debris due to gravitational interaction. For the new problems considered in the book, simplified and detailed mathematical models are developed, analytical solutions and estimates are obtained, new control laws are proposed, the results of numerical simulations are analyzed, regular and chaotic modes of the systems motions are studied. The book provides practical recommendations for the implementation of new pioneering missions related to the previously mentioned topics. It shows the reader new promising areas of research, opens up new scientific directions, and takes the first confident steps in each of them, providing a theoretical justification for the fundamental possibility of solving the new issues considered in the book.

The research findings presented in the book can serve as a foundation for groundbreaking space missions that hold significant practical and scientific value. In particular, the following missions can be mentioned:

- Study of the surface of Phobos and delivering samples to a spacecraft located at the libration point using a partial space elevator.

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- Long-term monitoring of Phobos' surface using a tether system attached to a spacecraft in a quasi-satellite orbit.
- Development of a transport system between Phobos and a quasi-satellite orbit using a tether system anchored on the Phobos' surface or fixed in the L_1 point.
- Using a space elevator anchored to the surface of Phobos to create a "stationary" probe between Mars and Phobos, to launch the probe on flyby trajectories around Mars, or on a landing trajectory to its surface.
- Delivering an electrostatic container from the surface of Phobos to a charged spacecraft located at the L_1 point by mechanically ejecting it from the surface and following capturing it due to Coulomb force. The presence of a charged spacecraft at the L_1 point and a container nearby leads to splitting the L_1 libration point and to the emergence of additional collinear unstable libration points.
- Collecting and transporting a cloud of space debris fragments from the geostationary orbit to a disposal orbit using a controlled artificial primary (gravity collector).
- New scheme of two-impulse transfer between a moon and its planet.

The numerical simulations included in the book demonstrate the feasibility of these missions and underline their advantages over traditional space technologies that rely on jet propulsion.

The author hopes that this book will be helpful for a wide range of scientists, engineers, graduate students, and students in the fields of mechanics and aerospace science, who are interested in the three-body problem, space debris problem, space tethered system dynamics, and contactless transportation in space.

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Moon-Anchored Space Elevators

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