Chaotic Behavior of a Passive Satellite During Towing by a Tether

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Outline

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Tug-Tether-Debris system

Debris with flexible appendages and/or fuel residuals

Tether (~$10^2$ m)

Tether control device

Tug
Space tether system (STS) – mechanical system of rigid bodies moving in different orbits, and the tethers (cables, ropes) that connect these bodies.

The system includes a space tug $C_1$, a viscoelastic tether $l$ and a space debris $C_2$.

The gravitational moment $M_g = 3m_0 l^2 \omega^2 \sin 2\alpha$ is gravitational moment.

$F$ is thrust.
The basic assumptions

1. An acceleration of the low-thrust tug is very small

\[ w_F = \frac{F}{m_1 + m_2} = g \]

then we can study an attitude motion of the system assuming that the orbit remains circular.

2. The tether is weightless

\[ m_t = 0 \]

3. Tether elongation and derivative of the elongation are small

\[ \Delta = \frac{l - l_0}{l_0} \ll 1, \quad \Delta' = \frac{d\Delta}{d\theta} \ll 1 \]

where \( \theta \) is true anomaly.
Mathematical model

Equation of the transverse oscillations of the tether

\[ \alpha'' + a \sin \alpha + b \sin 2\alpha = \Delta a \sin \alpha + 2 \frac{\Delta'}{(1+\Delta)} (\alpha' + 1) = \varepsilon f(\theta, \alpha, \alpha') \quad (1) \]

Equation of the longitudinal oscillations of the tether

\[ \Delta'' + \sigma^2 \Delta + 2\sigma\zeta\Delta' = a \cos \alpha + (1+\Delta)\alpha'^2 \quad (2) \]

where \((\ )' = d(\ )/d\theta\)

\[ a = F / (m_1 l_0 \omega^2), \quad b = 3, \quad \omega = \sqrt{\mu r^{-3}}, \quad \Omega = \sqrt{EA / m_0 l_0}, \]

\[ m_0 = m_1 m_2 / (m_1 + m_2), \quad \sigma = \Omega / \omega = \sqrt{EA r^3 / (m_0 l_0 \mu)}, \quad \varepsilon << 1 \]

where \(l_0\) is length of the unstretched tether, \(r\) is orbital radius, \(EA\) is a stiffness of the tether, \(\zeta\) is damping ratio of the tether.
The Equilibrium Positions

For the undisturbed motion \((\varepsilon = 0)\)

\[
\alpha'' - m_\alpha (\alpha) = 0 \quad \left[ m_\alpha (\alpha) = -(a \sin \alpha + b \sin 2\alpha) \right]
\]

Bifurcation diagram

If \(a = F / \left(m_1 l_0 \omega^2\right) < 6\) then

\[
\alpha_s = \pm \arccos \left( -\frac{a}{2b} \right)
\]

\(\alpha_u = 0\)

If \(a = F / \left(m_1 l_0 \omega^2\right) > 6\) then

\(\alpha_s = 0\)

and the unstable equilibrium position does not exist
The biharmonic moment

\[ m_\alpha (\alpha) = -(a \sin \alpha + b \sin 2\alpha) \]

The potential energy

\[ W(\alpha) = -\int m_\alpha (\alpha) d\alpha = -a \cos \alpha - b \cos^2 \alpha \]

The separatrices

\[ d\alpha / d\theta = S(\alpha) \]

for \( a = 5.96055, b = -3 \)
An approximate solution of Eq. (2) takes the form

\[ \Delta(\theta) = A \exp(-\sigma \zeta \theta) \sin(\lambda \theta + \phi_0) + C \]

Equation (1) describes the perturbed motion

\[ \alpha'' + a \sin \alpha + b \sin 2\alpha = \Delta a \sin \alpha + 2 \frac{\Delta'}{(1+\Delta)} (\alpha' + 1) \]

and if

\[ a = F \left( m_1 l_0 \omega^2 \right) < -2b = 6 \]

Then chaos becomes possible
Poincare sections

Poincaré surfaces of section are used to reveal the existence of a stochastic layer in the presence of damping of the tether

$$\zeta = 0.04$$

$$a = F / \left( m_1 l_0 \omega^2 \right) = 5.5 < 2b$$
Conclusion

The results of this study suggest that chaos can be observed for the motion of the tethered tug-debris system caused by longitudinal oscillations of the viscoelastic tether.

We have studied numerically the attitude motion dynamics by using construction of Poincare sections.

Furthermore, we have found the analytical criteria for determining the characteristics of the tethered system in which chaos does not exist.
Thank you

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