Attitude dynamics and control of space object during contactless transportation by ion beam

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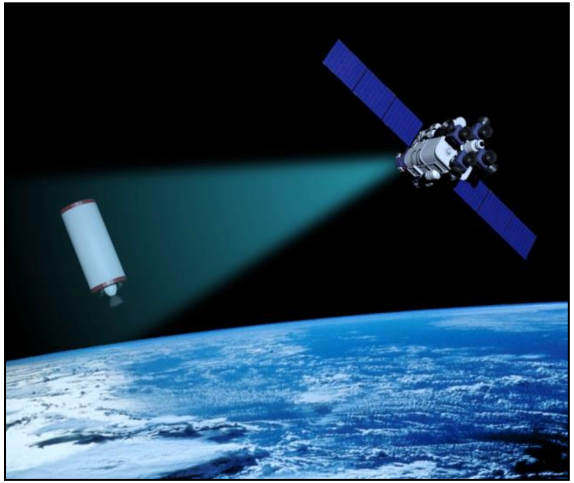
Dubai, 2020

Outline



- Introduction
- Mathematical model
- Equilibrium state
- Uncontrolled attitude motion
- Ion beam thrust control
- Results of numerical simulation
- Conclusions and results

Introduction

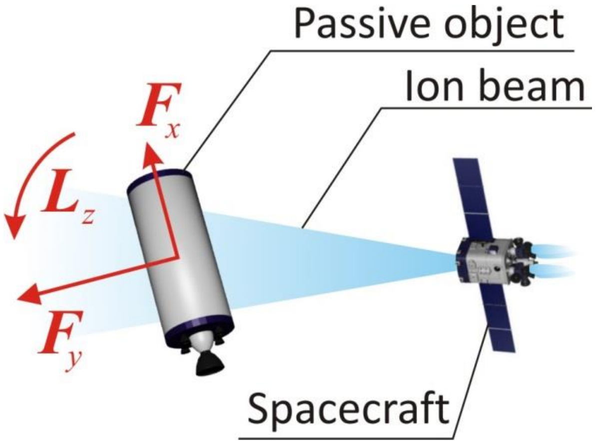
Application of contactless systems

- space debris removal
- serving long-lived satellites
- putting satellites into working orbits
- deflection of asteroids

Ion beam transportation systems

- ease of implementation,
- versatility
- safety

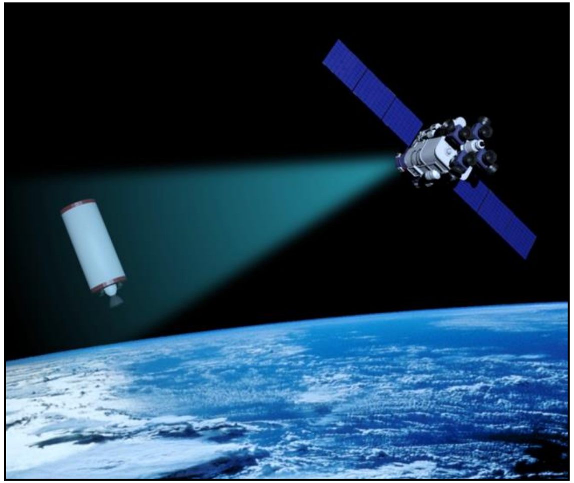
Introduction



The aims of this work

- study of a spatial motion dynamics of a passive space object under the influence of ion beam;
- development of the control law providing stabilization of the object oscillations.

Mathematical model. Assumptions



- Spacecraft is a mass point
- Passive object is a axisymmetric homogeneous cylinder
- Spacecraft's control system provides a constant distance between the object and the spacecraft
- Only ion and gravitational forces and torques act on the system

Mathematical model. Reference frames

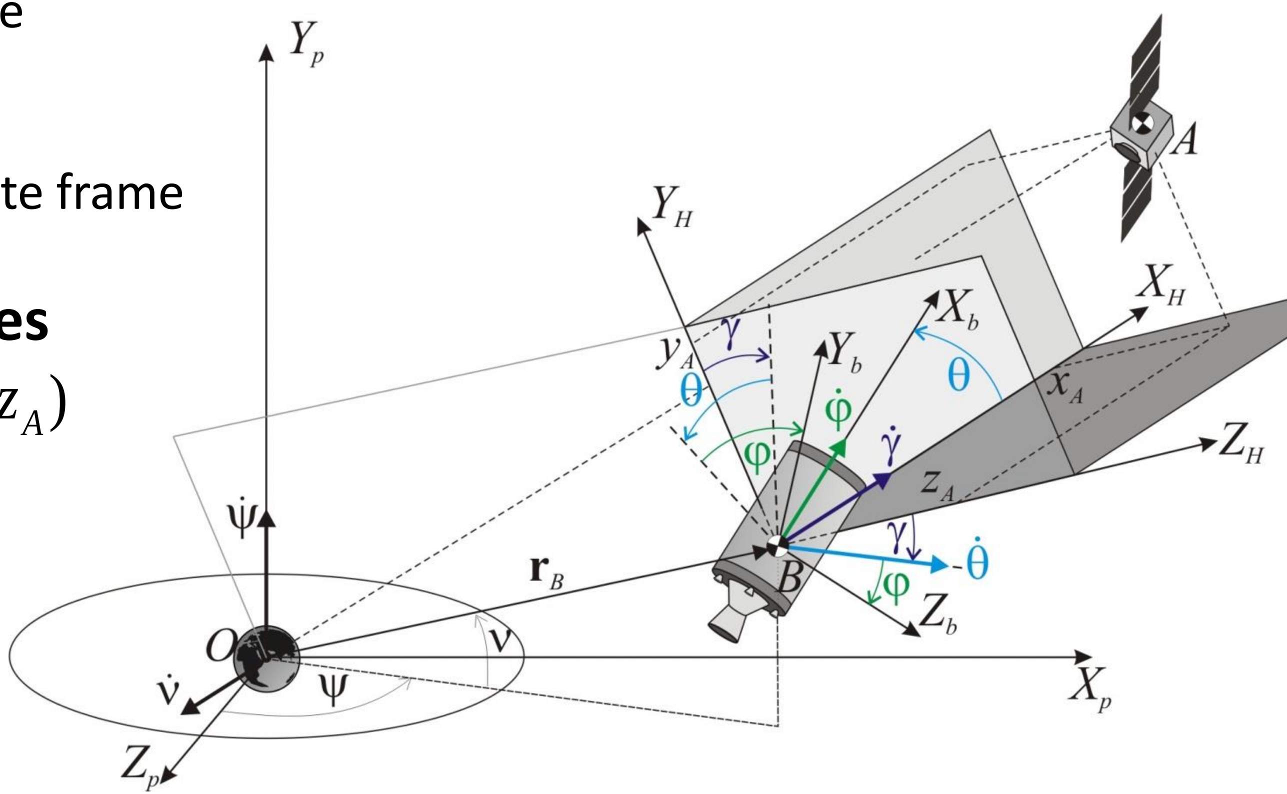
$OX_PY_PZ_P$ - inertial frame

$OX_bY_bZ_b$ - body frame

$OX_HY_HZ_H$ - Hill coordinate frame

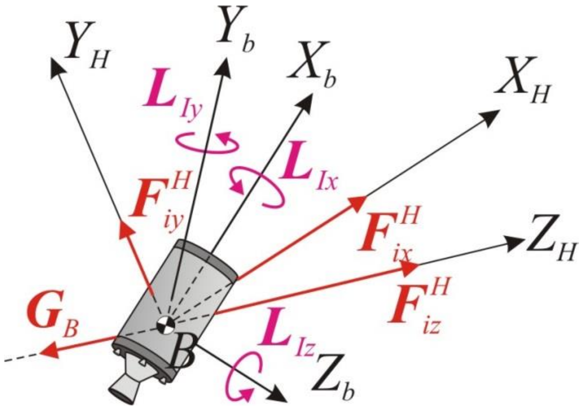
Generalized coordinates

$\mathbf{q} = (r, \nu, \psi, \gamma, \theta, \varphi, x_A, y_A, z_A)$



$$x_A = d = const, \quad y_A = 0, \quad z_A = 0$$

Object’s center of mass motion



$$\boxed{m_B \ddot{\mathbf{r}}^p = \mathbf{G}_B^p + \mathbf{F}_I^p} \qquad (1)$$

$$\mathbf{G}_B^p = -\mu m_B \mathbf{r}^p r^{-3}$$ Gravitational force

$$\mathbf{F}_i^H = [F_{ix}^H, F_{iy}^H, F_{iz}^H]^T$$ Ion beam force

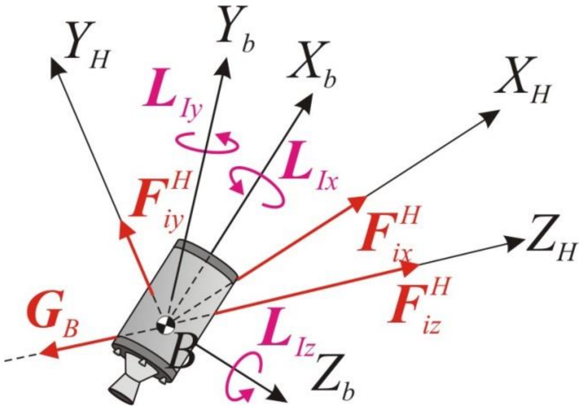
Equations of motion

$$\ddot{r} = r(\dot{v}^2 + \dot{\psi}^2 \cos v^2) - \frac{\mu}{r^2} + \frac{F_{iz}^H}{m_B}, \qquad (2)$$

$$\ddot{\psi} = \frac{2\dot{\psi}\dot{v}\sin v}{c\cos v} - \frac{2\dot{\psi}\dot{r}}{r} + \frac{F_{ix}^H}{r m_B \cos v}, \qquad (3)$$

$$\ddot{v} = -\dot{\psi}^2 \sin v \cos v - \frac{2\dot{v}\dot{r}}{r} + \frac{F_{iy}^H}{r m_B}. \qquad (4)$$

Object's attitude motion



$$\frac{d\mathbf{H}_B^b}{dt} + \boldsymbol{\omega}^b \times \mathbf{H}_B^b = \mathbf{L}_G^b + \mathbf{L}_I^b \qquad (5)$$

$\mathbf{H}_B^b = [\mathbf{I}]\boldsymbol{\omega}^b$ Angular momentum vector

$[\mathbf{I}]$ Object's inertia matrix

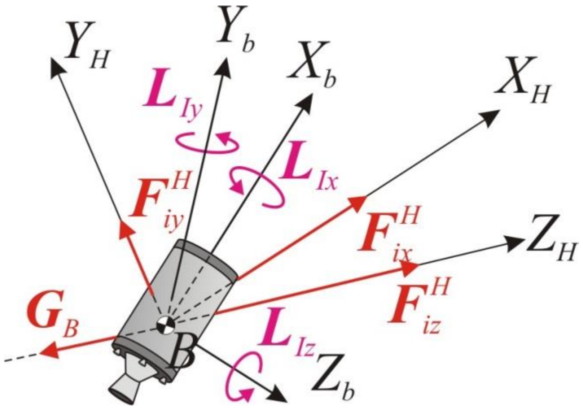
$\boldsymbol{\omega}^b = [\omega_x, \omega_y, \omega_z]^T$ Angular velocity

$\mathbf{L}_G^b = \frac{3\mu}{r^5}\mathbf{r}^b \times [\mathbf{I}]\mathbf{r}^b$ Gravity gradient torque

$\mathbf{L}_I^b = [L_{Ix}, L_{Iy}, L_{Iz}]^T$ Ion beam torque

$$\boldsymbol{\omega}^b = \dot{\mathbf{v}}^b + \dot{\boldsymbol{\psi}}^b + \dot{\boldsymbol{\gamma}}^b + \dot{\boldsymbol{\theta}}^b + \dot{\boldsymbol{\varphi}}^b \qquad (6)$$

Object's attitude motion in GEO



$$R = \bar{I}_x \omega_x, \quad G = R\cos\theta + (\omega_z \sin\varphi - \omega_y \cos\varphi)\sin\theta \qquad (7)$$

$$\bar{I}_x = I_x / I, \qquad I_y = I_z = I$$

$$\ddot{\theta} + \frac{(G - R\cos\theta)(R - G\cos\theta)}{\sin^3\theta} = \frac{L_{Iz}^{\varphi}(\theta)}{I} \qquad (8)$$

$$\dot{R} = L_{Ix}^{\varphi}(\theta) I^{-1} \qquad (9)$$

$$\dot{G} = L_{Ix}^{\varphi}(\theta) I^{-1} \cos\theta - L_{Iy}^{\varphi}(\theta) I^{-1} \sin\theta \qquad (10)$$

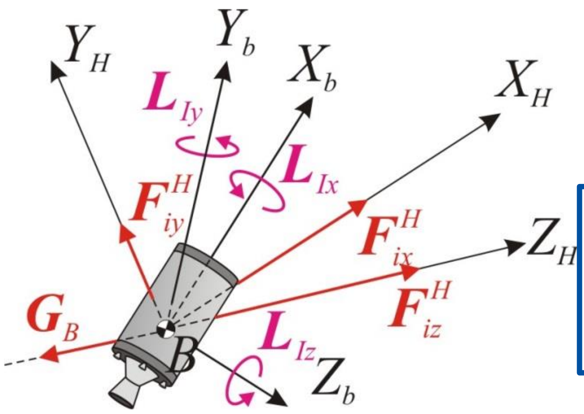
$$\dot{\gamma} = (G - R\cos\theta)\sin^{-2}\theta \qquad (11)$$

$$\dot{\varphi} = R\bar{I}_x^{-1} - (G - R\cos\theta)\cos\theta\sin^{-2}\theta \qquad (12)$$

$$L_{Ix}^{\varphi} = L_{Ix}, \quad L_{Iy}^{\varphi} = L_{Iy}\cos\varphi - L_{Iz}\sin\varphi,$$

$$L_{Iz}^{\varphi} = L_{Iy}\sin\varphi + L_{Iz}\cos\varphi$$

Object's attitude motion in GEO



$$L_{Ix}^{\varphi} = L_{Iy}^{\varphi} = 0, \qquad L_{Iz}^{\varphi} = uL_I^{\max} \sum_j^k b_j \sin j\theta \tag{13}$$

$u \in [0,1]$ Control parameter

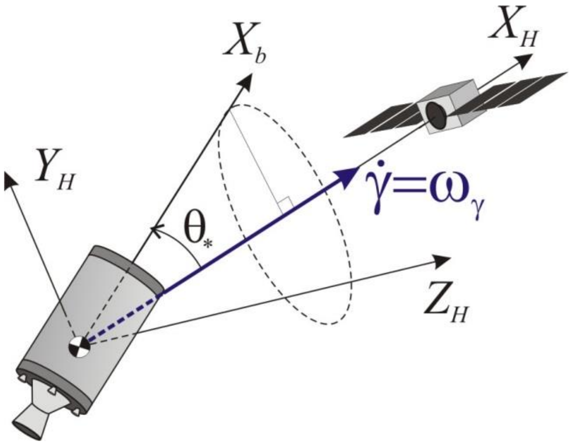
$$\ddot{\theta} + \frac{(G - R\cos\theta)(R - G\cos\theta)}{\sin^3\theta} = \frac{uL_I^{\max}}{I} \sum_j^k b_j \sin j\theta \tag{14}$$

$$R = const, \quad G = const$$

$$\frac{\dot{\theta}^2}{2} + W(\theta) = E \tag{15}$$

$$W(\theta) = \frac{G^2 + R^2 - 2GR\cos\theta}{2\sin^2\theta} + \frac{uL_I^{\max}}{I} \sum_j^k \frac{b_j}{j} \cos j\theta \tag{16}$$

Object's equilibrium state



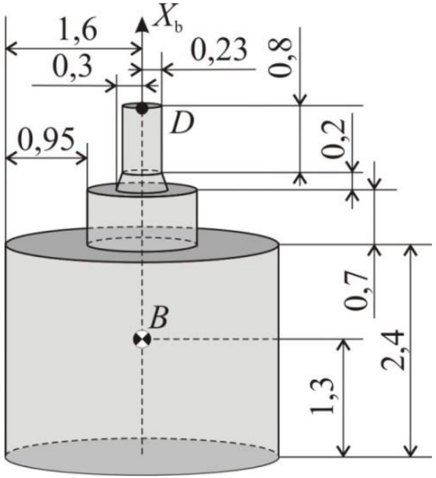
$$0+\frac{(G_*-R_*\cos\theta_*)(R_*-G_*\cos\theta_*)}{\sin^3\theta_*}=\frac{L_{Iz}^{\varphi}(\theta_*)}{I} \qquad (8)$$

$$0=\frac{G_*-R_*\cos\theta_*}{\sin^2\theta_*} \qquad (11)$$

$$\theta_*=\arccos(G_*/R_*)$$
$$L_{Iz}^{\varphi}(\theta_*)=0 \qquad (17)$$

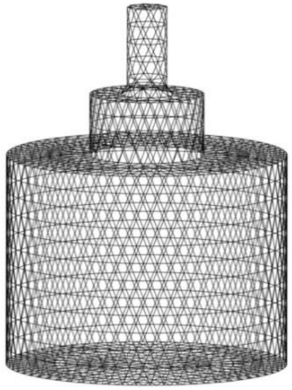
$$\theta_*\neq\arccos(G_*/R_*) \qquad \dot{\gamma}=\frac{G_*-R_*\cos\theta_*}{\sin^2\theta_*}=\omega_\gamma=const$$

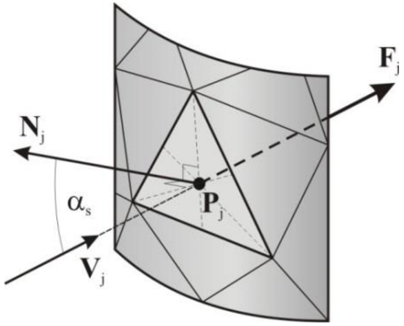
System parameters

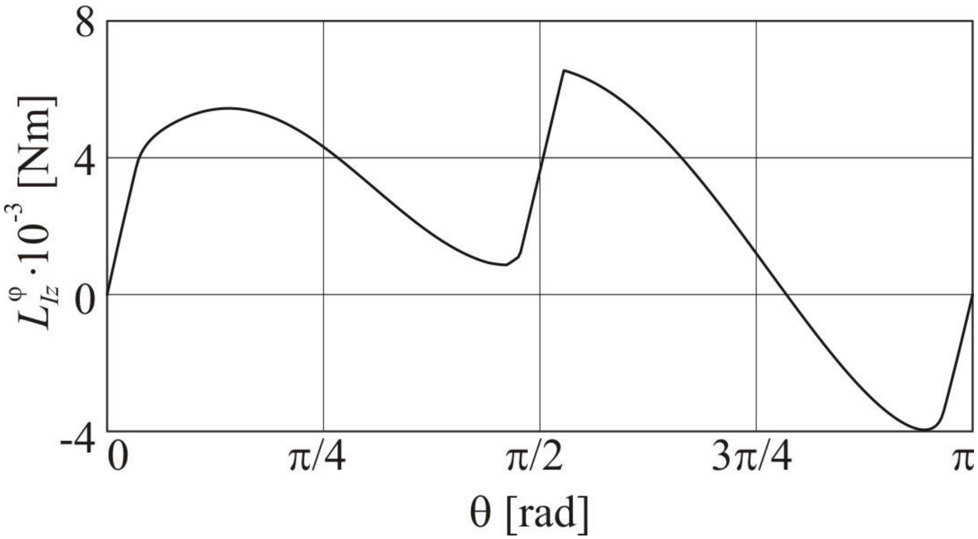


Parameter	Value
Mass m_B	1100 kg
Moment of inertia I_x	1400 kg m^2
Moments of inertia I_y, I_z	2100 kg m^2
Plasma density n_0	2.6 10^{16} m^{-3}
Mass of the particle(xenon) m_0	2.18 10^{-25} kg
Ion velocity u_0	38000 m/s
Ion beam divergence angle	15 deg
Distance d	15 m

Ion beam torque



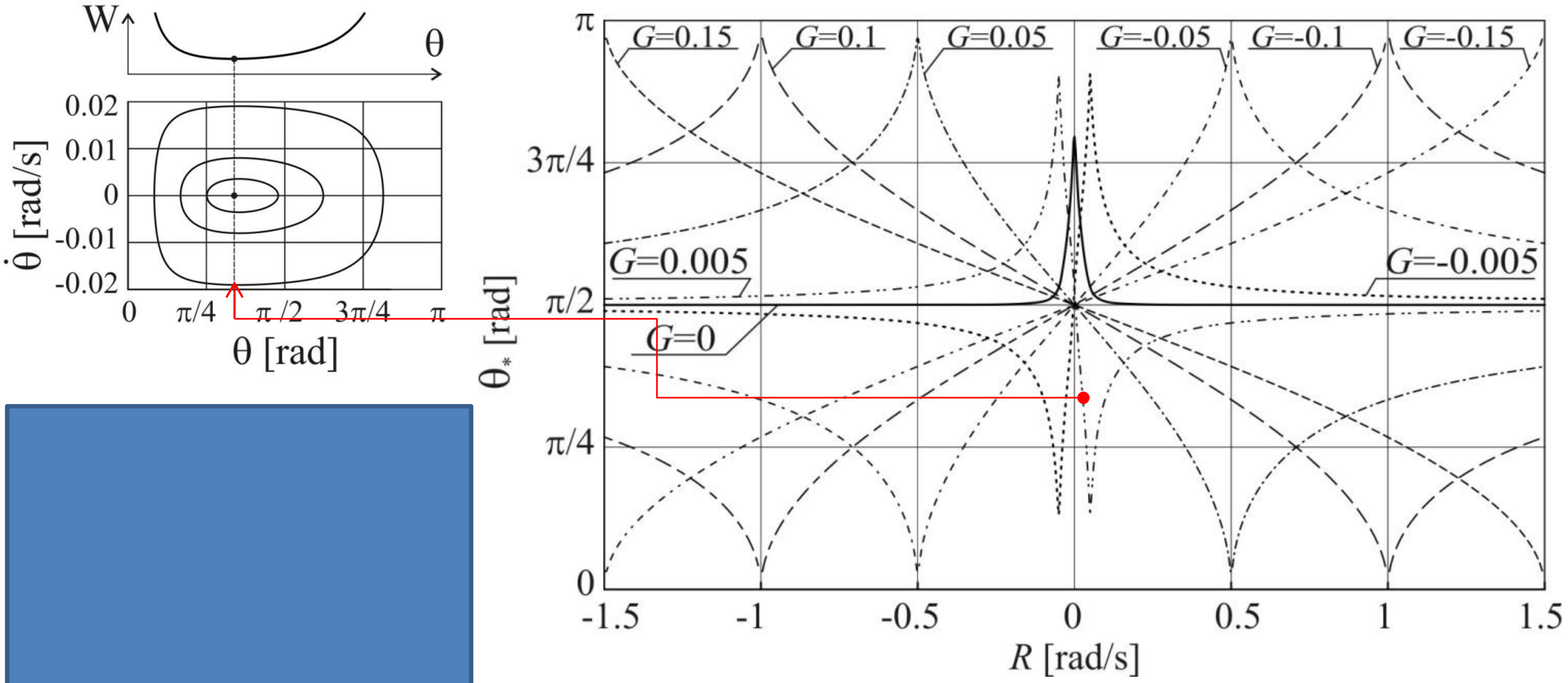




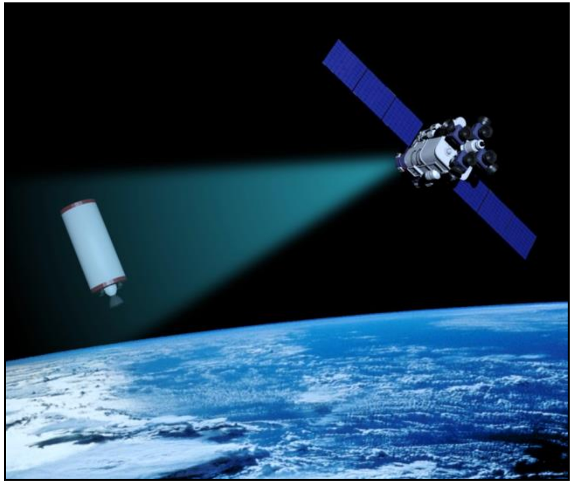
[1] A.P. Alpatov, S.V. Khoroshylov, A.I. Maslova, Contactless de-orbiting of space debris by the ion beam. Dynamics and Control. — Kyiv: Akademperiodyka, 2019. — 170 p. DOI: 10.15407/akademperiodyka.383.170. (Chapter 3)

[2] V.S. Aslanov, A.S. Ledkov, Attitude motion of cylindrical space debris during its removal by ion beam, Mathematical Problems in Engineering. (2017) Article ID 1986374. DOI: 10.1155/2017/1986374.

Uncontrolled attitude motion



Ion beam thrust control



$$L_{Iz}^{\varphi} = uL_I^{\max}\sum_j^k b_j \sin j\theta$$

$$u = \begin{cases} 1 + k(\theta - \theta_*)\dot{\theta}\,\mathrm{H}[(\theta_* - \theta)\dot{\theta}], & \text{when } k(\theta - \theta_*)\dot{\theta} > -1; \\ 1, & \text{when } k(\theta - \theta_*)\dot{\theta} \le -1; \end{cases}$$

$H[]$ Heaviside theta function

$$k = \frac{1}{\max\limits_{[t-T,t]}(|(\theta - \theta_*)\dot{\theta}|)}$$

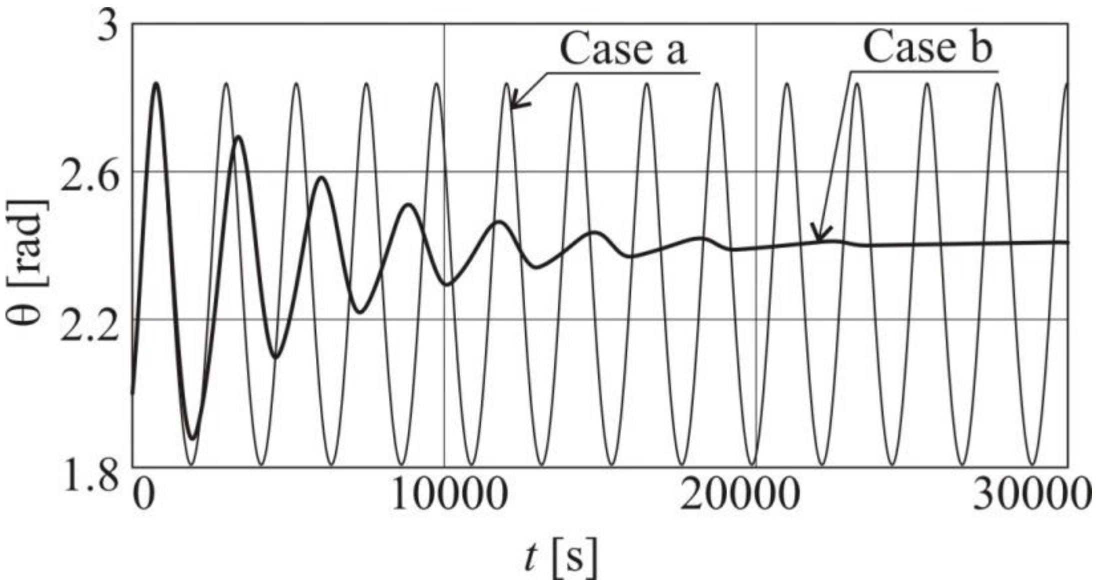
Control law parameter

Numerical simulation result

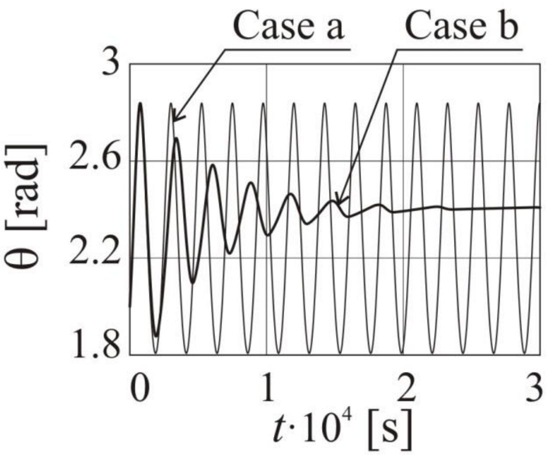
Initial conditions

$$\omega_x = 0.001\,\text{rad/s},\ \omega_y = \omega_z = 0,\ \theta_0 = 2\text{rad},\ \gamma_0 = \varphi_0 = 0,\ \dot{\theta}_0 = 0.001\,\text{rad/s},$$

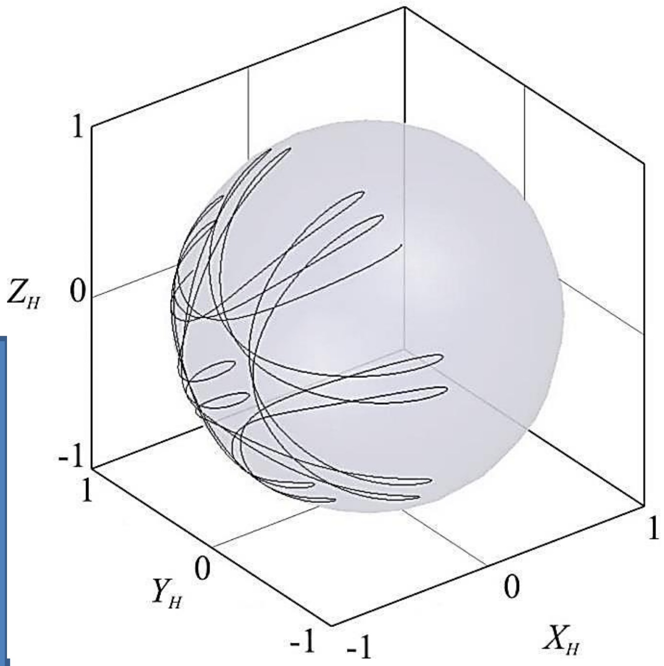
$$R = 6.6667 \cdot 10^{-4}\ \text{rad/s},\ G = \text{-}2.7743 \cdot 10^{-4}\ \text{rad/s},\ \theta_* = 2.4082\,\text{rad}$$



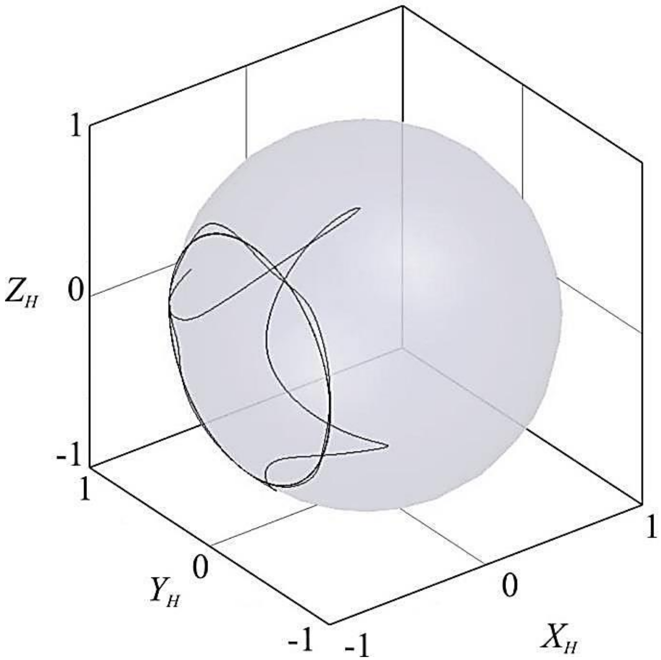
Numerical simulation result



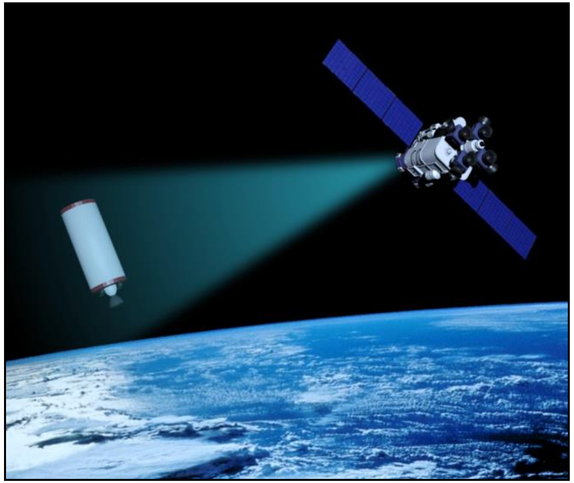
Case a



Case b



Conclusions and results



- A mathematical model describing the spatial motion of a passive space object relative its center of mass was developed.
- Possible equilibrium positions was found.
- The ion beam thrust control law, which provides stabilization of the spatial oscillations of the object in the equilibrium position, was proposed.
- The effectiveness of the law was confirmed by the results of numerical simulations.

Thank you!

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This study was supported by the grant in form of subsidies from the federal budget, allocated for state support of scientific research under supervision of leading scientists in Russian institutions of higher education, scientific foundations and state research centers of the Russian Federation (7th stage, Decree of the Government of the Russian Federation No. 220 of 09 April 2010), **project No. 075-15-2019-1894.**