Influence of space debris attitude motion on ion beam assisted removal mission costs

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Outline

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- Mathematical model
- Space debris unperturbed motion
- Average ion beam force calculation
- Control laws
- Results of numerical simulation
- Conclusions and results
Introduction

34000 objects >10 cm
5400 objects >1m
2000 active satellites

Active space debris removal approaches:
• docking or hard grip of an object
• capturing and tethered towing
• contactless transportation

Contactless interaction
• electrostatic
• gravitational
• magnetic
• laser irradiation
• ion flow blowing
Idea authors:
- C. Bombardelli and J. Pelaez
  (Ion Beam Shepherd)
- S. Kitamura
  (Ion Beam Irradiation Reorbiter)


The aim and objectives

The aim is to study the effect of space debris attitude motion on removal mission costs.

Objectives

- mathematical model development
- study of an unperturbed motion dynamics in a circular orbit
- determination of favorable angular motion modes of the space debris
- numerical simulation and analysis of the space debris removal mission
Mathematical model

Assumptions

• Planar motion is considered
• Space debris and spacecraft are rigid bodies
• Space debris is an axisymmetric cylinder
• Only ion and gravitational forces and torques act on the system
• The gravitational field is Newtonian
Mathematical model

Lagrange equations
\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \]

Generalized coordinates
- \( r \) - position vector length
- \( \nu \) - true anomaly angle
- \( \theta \) - space debris deflection angle
- \( \alpha \) - angle between \( Ay \) axis and \( AB \)
- \( d \) - distance between centers of mass \( A \) and \( B \)
- \( \beta \) – ion beam axis deflection angle
Mathematical model

**Lagrange equations**

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i
\]

**Lagrange function**

\[
L = \frac{m_A V_A^2}{2} + \frac{I_{A_z} (\ddot{\gamma} + \dot{\alpha} + \dot{\beta})^2}{2} + \frac{m_B V_B^2}{2} + \frac{I_{B_z} (\ddot{\gamma} + \dot{\alpha} + \dot{\theta})^2}{2}
\]

\[
\begin{align*}
\mu m_A &+ \frac{\mu(I_{Ax} + I_{Ay} + I_{Az})}{r_A^3} - \frac{3\mu(I_{Ax} \cos^2 \gamma_A + I_{Ay} \sin^2 \gamma_A + I_{Az})}{2r_A^3} \\
\mu m_B &+ \frac{\mu(I_{Bx} + I_{By} + I_{Bz})}{r_B^3} - \frac{3\mu(I_{Bx} \cos^2 \gamma_B + I_{By} \sin^2 \gamma_B + I_{Bz})}{2r_B^3}
\end{align*}
\]

\[\eta = \arctan \left( \frac{d \cos \alpha}{r - d \sin \alpha} \right), \quad \gamma_A = \alpha + \beta, \quad \gamma_B = \alpha + \theta - \eta\]
Lagrange equations

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \]

Generalized forces

\[ Q_r = P_x + F_x \cos(\alpha + \theta) - F_y \sin(\alpha + \theta) \]
\[ Q_v = L_z + P_y r + F_x (r \sin(\alpha + \theta) - d \cos \theta) \]
\[ + F_y (r \cos(\alpha + \theta) + d \sin \theta) \]
\[ Q_\theta = L_z \quad Q_\beta = M_z \]
\[ Q_\alpha = -F_x d \cos \theta + F_y d \sin \theta + L_z \]
\[ Q_d = F_x \sin \theta + F_y \cos \theta \]
### System parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacecraft mass $m_A$</td>
<td>500 kg</td>
</tr>
<tr>
<td>Space debris mass $m_B$</td>
<td>1400 kg</td>
</tr>
<tr>
<td>Space debris length</td>
<td>6.5 m</td>
</tr>
<tr>
<td>Space debris radius</td>
<td>1.2 m</td>
</tr>
<tr>
<td>Space debris moment of inertia $I_x$</td>
<td>1300 kg m$^2$</td>
</tr>
<tr>
<td>Space debris moments of inertia $I_y, I_z$</td>
<td>6000 kg m$^2$</td>
</tr>
<tr>
<td>Spacecraft moments of inertia $I_x, I_y, I_z$</td>
<td>400 kg m$^2$</td>
</tr>
<tr>
<td>Plasma density $n_0$</td>
<td>$2.6 \times 10^{16}$ m$^{-3}$</td>
</tr>
<tr>
<td>Mass of the particle (xenon) $m_0$</td>
<td>$2.18 \times 10^{-25}$ kg</td>
</tr>
<tr>
<td>Ion velocity $u_0$</td>
<td>38000 m/s</td>
</tr>
<tr>
<td>Ion beam divergence angle</td>
<td>15 deg</td>
</tr>
</tbody>
</table>
Ion beam forces and torques

\[ F_x = F_x(d, \alpha, \beta, \theta) \quad F_y = F_y(d, \alpha, \beta, \theta) \quad L_z = L_z(d, \alpha, \beta, \theta) \]


Space debris unperturbed motion

\[ r = \text{const}, \quad \dot{r} = \omega = \sqrt{\mu r^{-3}}, \quad d = \text{const}, \quad \alpha = 0, \quad \beta = 0 \]

\[ \ddot{\theta} = \frac{L_{Bz}(\theta)}{I_z} - \frac{3\mu(I_{By} - I_{Bx})}{2r^3 I_{Bz}} \sin 2\theta \]

Energy integral

\[ E = \frac{\dot{\theta}^2}{2} - \frac{\int L_z(\theta)d\theta}{I_z} - \frac{3\mu(I_{By} - I_{Bx})}{4r^3 I_{Bz}} \cos 2\theta \]
Average ion beam force

\[ F_D = F_x \sin \theta + F_y \cos \theta \]
Spacecraft control laws

Relative position of the spacecraft

\[ M_\alpha = (\alpha - 0)k_{\alpha 1} + \dot{\alpha}k_{\alpha 2} \quad \text{and} \quad F_d = (d - d_0)k_{d 1} + \dot{d}k_{d 2} \]

\[ P_x = -\frac{M_\alpha}{d} \cos \alpha - F_d \sin \alpha \quad \text{and} \quad P_y = -\frac{M_\alpha}{d} \sin \alpha + F_d \cos \alpha \]

Direction of the ion beam axis

\[ M_z = (\beta_0 - \beta)k_{\beta 1} - \dot{\beta}k_{\beta 2} \]

\( L_z < 0 \) and \( L_z > 0 \)
Space debris control

\[
M_z = \begin{cases} 
(\beta_1 - \beta)k_1\beta - \dot{\beta}k_2\beta, & \text{when } L_z(\theta, \beta_1)\dot{\theta}(E_* - E) > 0, \\
(\beta_2 - \beta)k_1\beta - \dot{\beta}k_2\beta, & \text{when } L_z(\theta, \beta_2)\dot{\theta}(E_* - E) > 0, \\
-\beta k_1\beta - \dot{\beta}k_2\beta & \text{otherwise}
\end{cases}
\]
Results of numerical simulation

\[ \theta, \text{ rad} \]

\[ d, \text{ m} \]

\[ \alpha, \beta, \text{ rad} \]

\[ P_x, P_y, \text{ N} \]

\[ M_z, \text{ Nm} \]
Results of numerical simulation

\( \beta, \text{ rad} \)

\( t, \text{s} \)

\( M_2, \text{ Nm} \)

\( t, \text{s} \)

\( \theta, \text{ rad} \)

\( \dot{\theta}, \text{ rad/s} \)
Results of numerical simulation

\[
\dot{m} = \frac{T}{I_{sp} g_0}
\]

\[I_{sp} = 2000\text{s}\]

\[
m_1 = 11.1648\text{kg}
\]

\[
m_2 = 9.3418\text{kg}
\]

\[
\Delta t = 98.76\text{ hours}
\]

\[
\frac{m_1 - m_2}{m_2} = 0.1633
\]
Conclusions and results

• The mathematical model was developed using the Lagrange formalism.
• The undisturbed oscillations of the cylindrical space debris were studied.
• A phase trajectory on which the average ion beam force is maximum in absolute value was determined.
• The control law of the spacecraft orientation engines, which ensures the transfer of the space debris object into a motion along the phase trajectory with maximum average ion beam drag force, was proposed.
• It was shown that the attitude motion of a spacecraft during transportation has a significant effect on the required fuel costs.
Thank you!

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