Influence of space debris attitude motion on ion beam assisted removal mission costs

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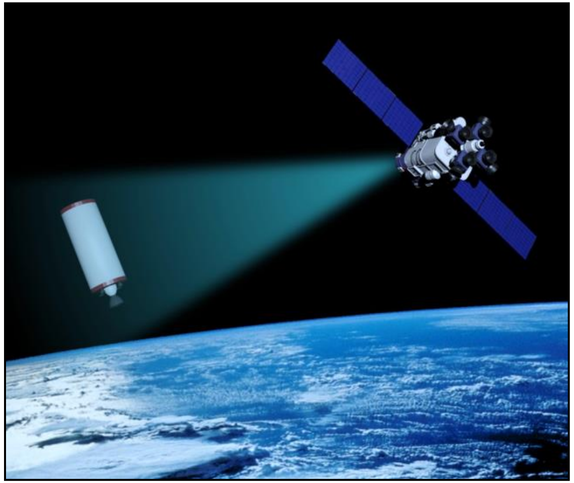
Outline



- Introduction
- Mathematical model
- Space debris unperturbed motion
- Average ion beam force calculation
- Control laws
- Results of numerical simulation
- Conclusions and results

Introduction

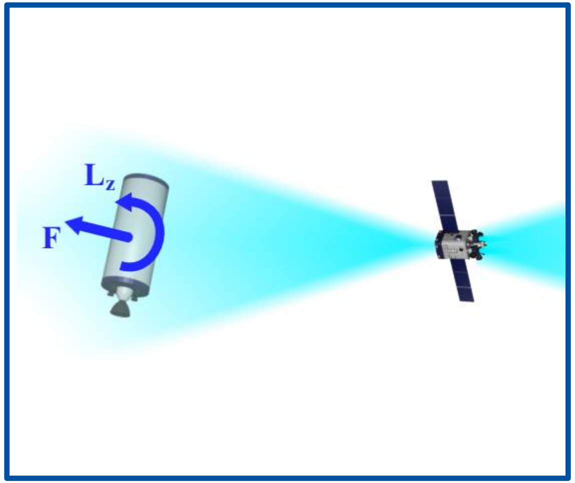
Active space debris removal approaches:

- docking or hard grip of an object
- capturing and tethered towing
- contactless transportation

Contactless interaction

- electrostatic
- gravitational
- magnetic
- laser irradiation
- ion flow blowing

Ion beam transportation



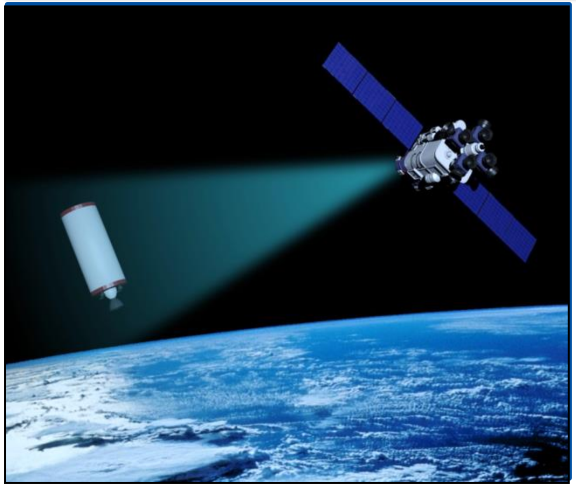
Idea authors:

- C. Bombardelli and J. Pelaez
  (Ion Beam Shepherd)
- S. Kitamura
  (Ion Beam Irradiation Reorbiter)

[1] Bombardelli C., Pelaez J. Sistema de modificación de la posición y actitud de cuerpos en órbita por medio de satélites guía, *Patent* No. P201030354, filed 11 March 2010

[2] Kitamura, S., Large Space Debris reorbiter using ion beam irradiation, *61st International Astronautical Congress*, International Astronautical Federation, Paris, France, 2010.

The aim and objectives

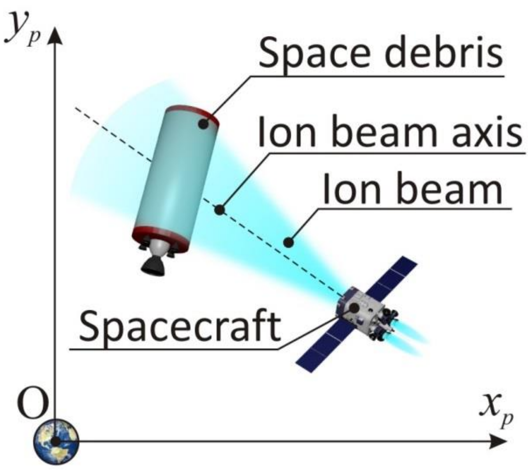


The **aim** is to study the effect of space debris attitude motion on removal mission costs.

Objectives

- mathematical model development
- study of an unperturbed motion dynamics in a circular orbit
- determination of favorable angular motion modes of the space debris
- numerical simulation and analysis of the space debris removal mission

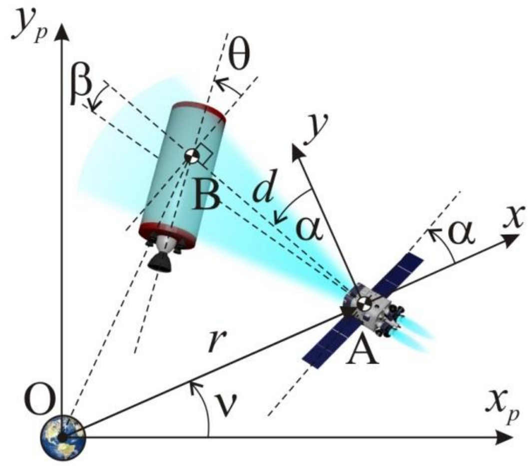
Mathematical model



Assumptions

- Planar motion is considered
- Space debris and spacecraft are rigid bodies
- Space debris is an axisymmetric cylinder
- Only ion and gravitational forces and torques act on the system
- The gravitational field is Newtonian

Mathematical model



Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

Generalized coordinates

r - position vector length

ν - true anomaly angle

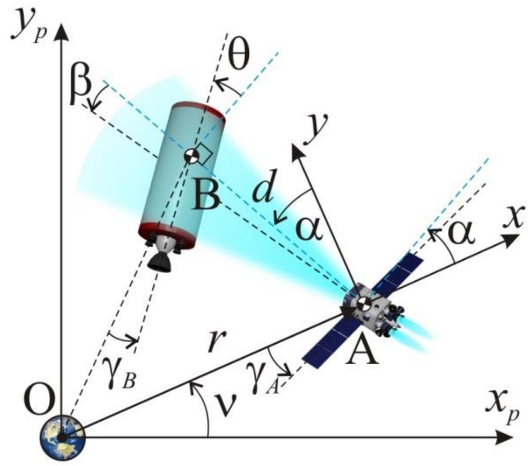
θ - space debris deflection angle

α - angle between Ay axis and AB

d - distance between centers of mass A and B

β – ion beam axis deflection angle

Mathematical model



Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}-\frac{\partial L}{\partial q_i}=Q_i$$

Lagrange function

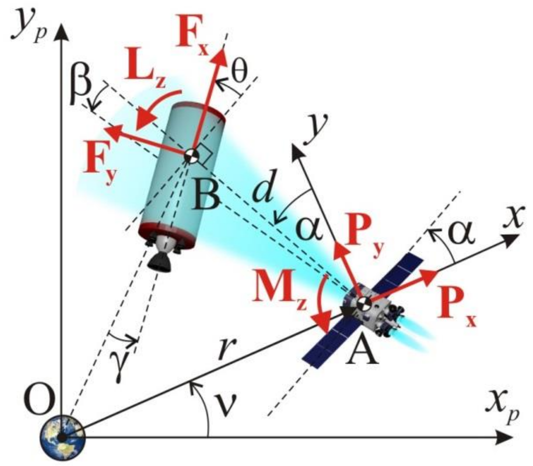
$$L=\boxed{\frac{m_A V_A^2}{2}+\frac{I_{Az}(\dot{\nu}+\dot{\alpha}+\dot{\beta})^2}{2}}+\boxed{\frac{m_B V_B^2}{2}+\frac{I_{Bz}(\dot{\nu}+\dot{\alpha}+\dot{\theta})^2}{2}}$$

$$+\boxed{\frac{\mu m_A}{r_A}+\frac{\mu(I_{Ax}+I_{Ay}+I_{Az})}{2r^3}-\frac{3\mu(I_{Ax}\cos^2\gamma_A+I_{Ay}\sin^2\gamma_A+I_{Az})}{2r^3}}$$

$$+\boxed{\frac{\mu m_B}{r_B}+\frac{\mu(I_{Bx}+I_{By}+I_{Bz})}{2r_B^3}-\frac{3\mu(I_{Bx}\cos^2\gamma_B+I_{By}\sin^2\gamma_B+I_{Bz})}{2r_B^3}}$$

$$\eta=\arctan\left(\frac{d\cos\alpha}{r-d\sin\alpha}\right),\quad \gamma_A=\alpha+\beta,\quad \gamma_B=\alpha+\theta-\eta$$

Mathematical model



Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

Generalized forces

$$Q_r = P_x + F_x \cos(\alpha+\theta) - F_y \sin(\alpha+\theta)$$

$$Q_\nu = L_z + P_y r + F_x (r \sin(\alpha+\theta) - d\cos\theta) + F_y (r\cos(\alpha+\theta) + d\sin\theta)$$

$$Q_\theta = L_z \qquad Q_\beta = M_z$$

$$Q_\alpha = -F_x d\cos\theta + F_y d\sin\theta + L_z$$

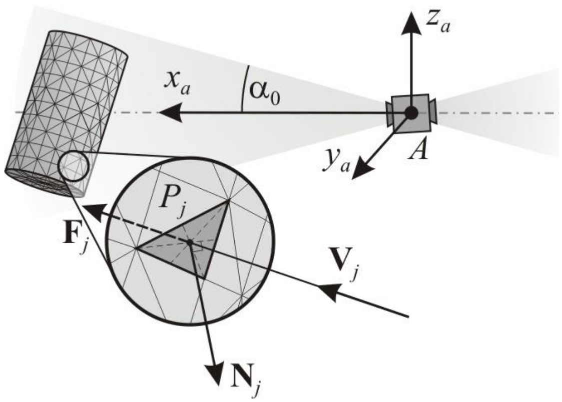
$$Q_d = F_x \sin\theta + F_y \cos\theta$$

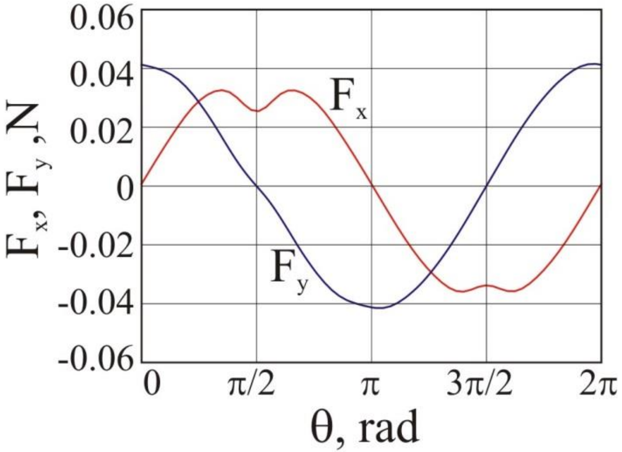
System parameters

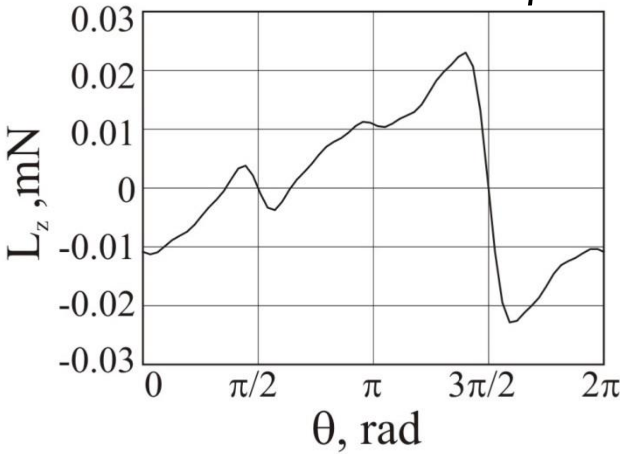
Parameter	Value
Spacecraft mass m_A	500 kg
Space debris mass m_B	1400 kg
Space debris length	6.5 m
Space debris radius	1.2 m
Space debris moment of inertia I_x	1300 kg m^2
Space debris moments of inertia I_y, I_z	6000 kg m^2
Spacecraft moments of inertia I_x, I_y, I_z	400 kg m^2
Plasma density n_0	$2.6\ 10^{16}$ m^{-3}
Mass of the particle(xenon) m_0	$2.18\ 10^{-25}$ kg
Ion velocity u_0	38 000 m/s
Ion beam divergence angle	15 deg

Ion beam forces and torques

d=15m α=0 β=0







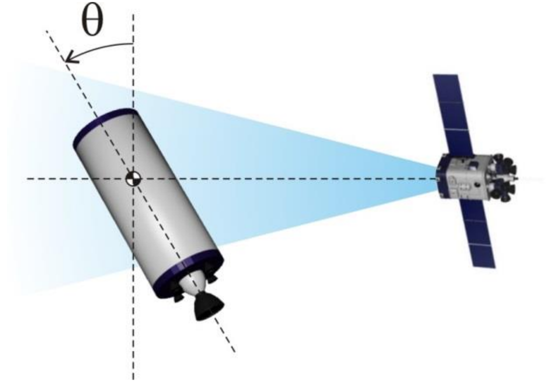
$$F_x = F_x(d,\alpha,\beta,\theta) \quad F_y = F_y(d,\alpha,\beta,\theta) \quad L_z = L_z(d,\alpha,\beta,\theta)$$

[3] A.P. Alpatov, S.V. Khoroshylov, A.I. Maslova, Contactless de-orbiting of space debris by the ion beam. Dynamics and Control. — Kyiv: Akademperiodyka, 2019. — 170 p. DOI: 10.15407/akademperiodyka.383.170. (Chapter 3)

[4] V.S. Aslanov, A.S. Ledkov, Attitude motion of cylindrical space debris during its removal by ion beam, Mathematical Problems in Engineering. (2017) Article ID 1986374. DOI: 10.1155/2017/1986374.

Space debris unperturbed motion

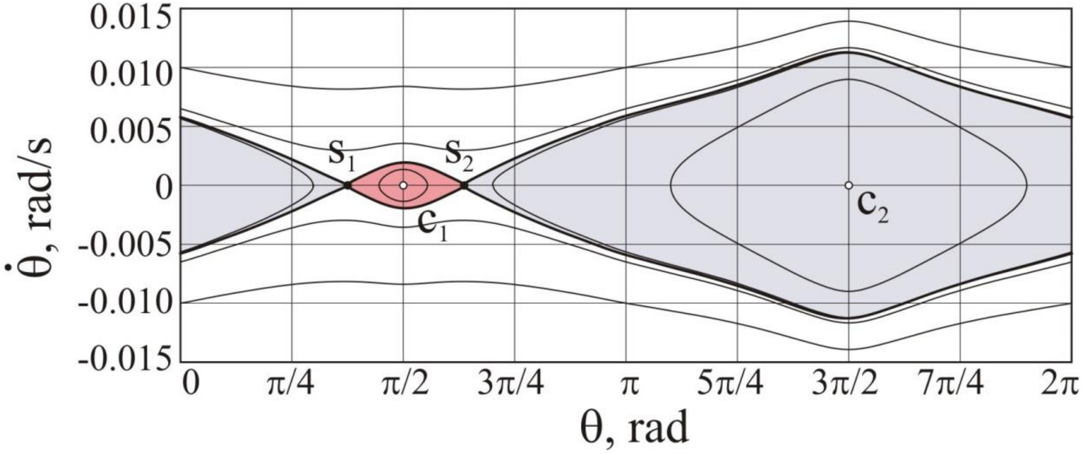
$$r = const, \quad \dot{\nu} = \omega = \sqrt{\mu r^{-3}}, \quad d = const, \quad \alpha = 0, \quad \beta = 0$$



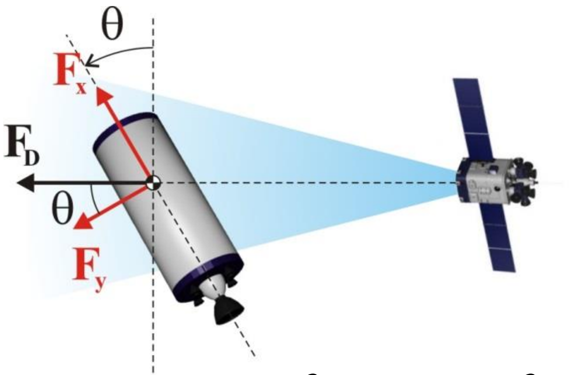
$$\boxed{\ddot{\theta} = \frac{L_{Bz}(\theta)}{I_z} - \frac{3\mu(I_{By} - I_{Bx})}{2r^3 I_{Bz}} \sin 2\theta}$$

Energy integral

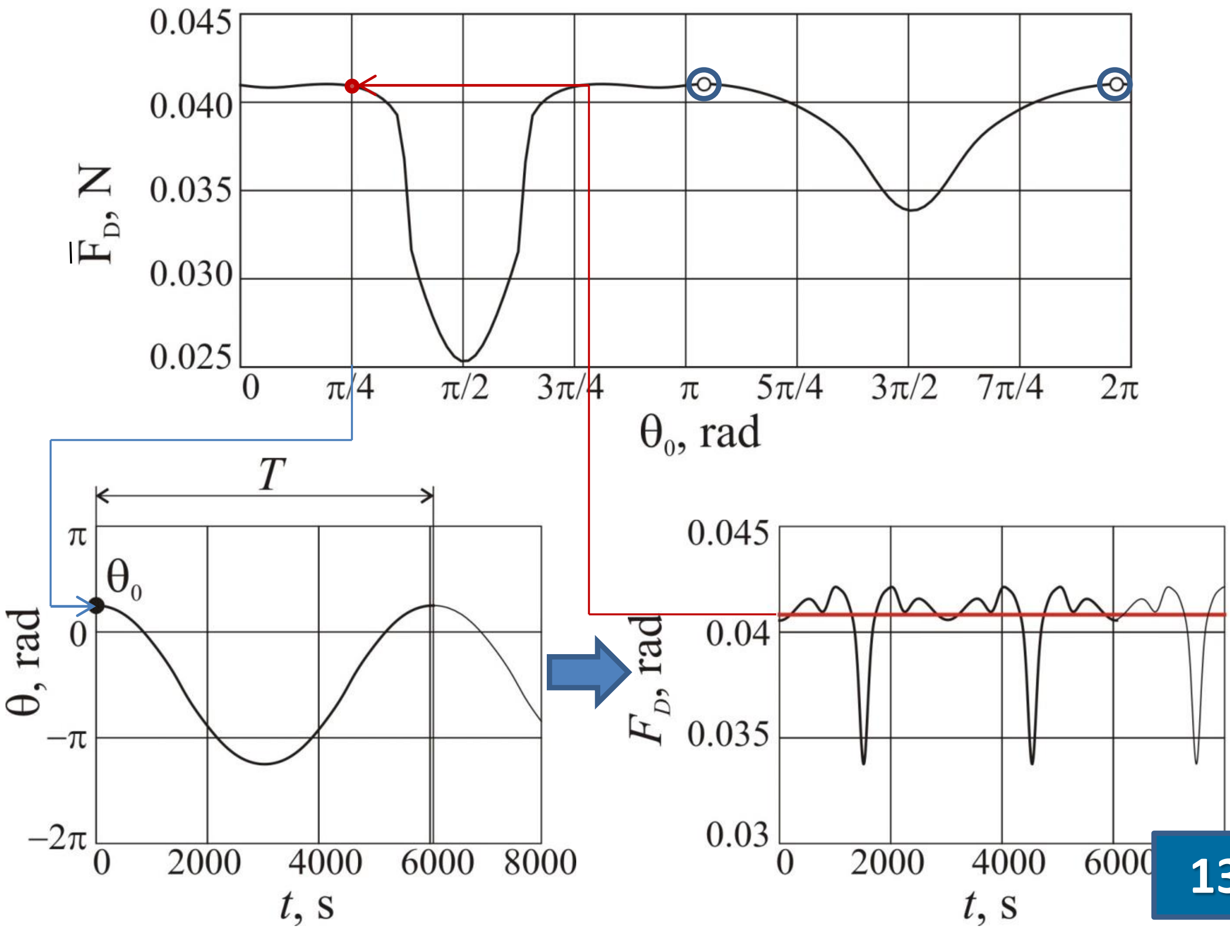
$$\boxed{E = \frac{\dot{\theta}^2}{2} - \frac{\int L_z(\theta) d\theta}{I_z} - \frac{3\mu(I_{By} - I_{Bx})}{4r^3 I_{Bz}} \cos 2\theta}$$



Average ion beam force



$$F_D = F_x \sin\theta + F_y \cos\theta$$



Spacecraft control laws

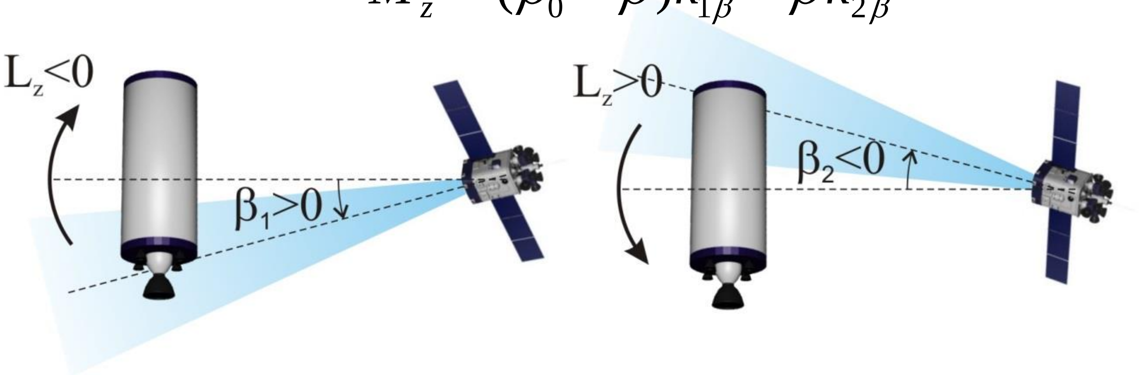
Relative position of the spacecraft

$$M_{\alpha} = (\alpha - 0)k_{\alpha 1} + \dot{\alpha} k_{\alpha 2} \qquad F_d = (d - d_0)k_{d1} + \dot{d}\, k_{d2}$$

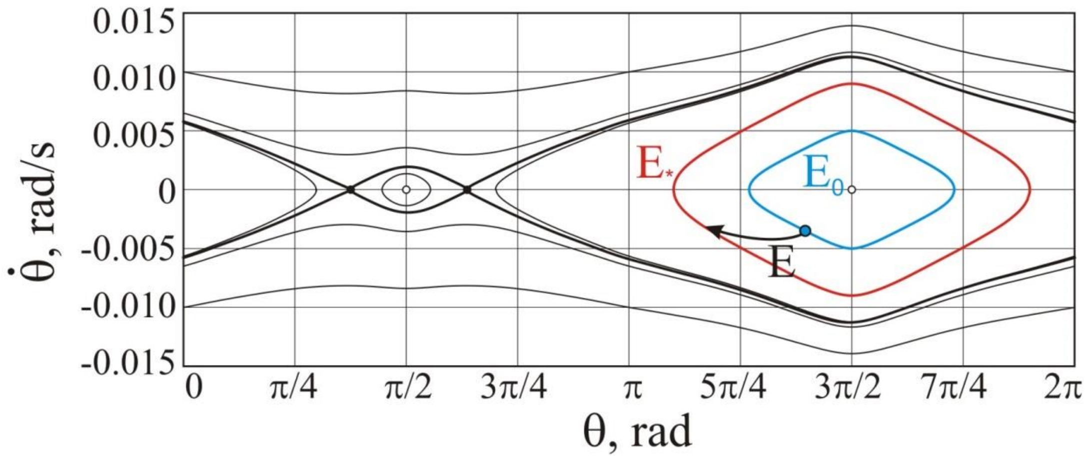
$$P_x = -\frac{M_{\alpha}}{d}\cos\alpha - F_d \sin\alpha \qquad P_y = -\frac{M_{\alpha}}{d}\sin\alpha + F_d \cos\alpha$$

Direction of the ion beam axis

$$M_z = (\beta_0 - \beta)k_{1\beta} - \dot{\beta} k_{2\beta}$$

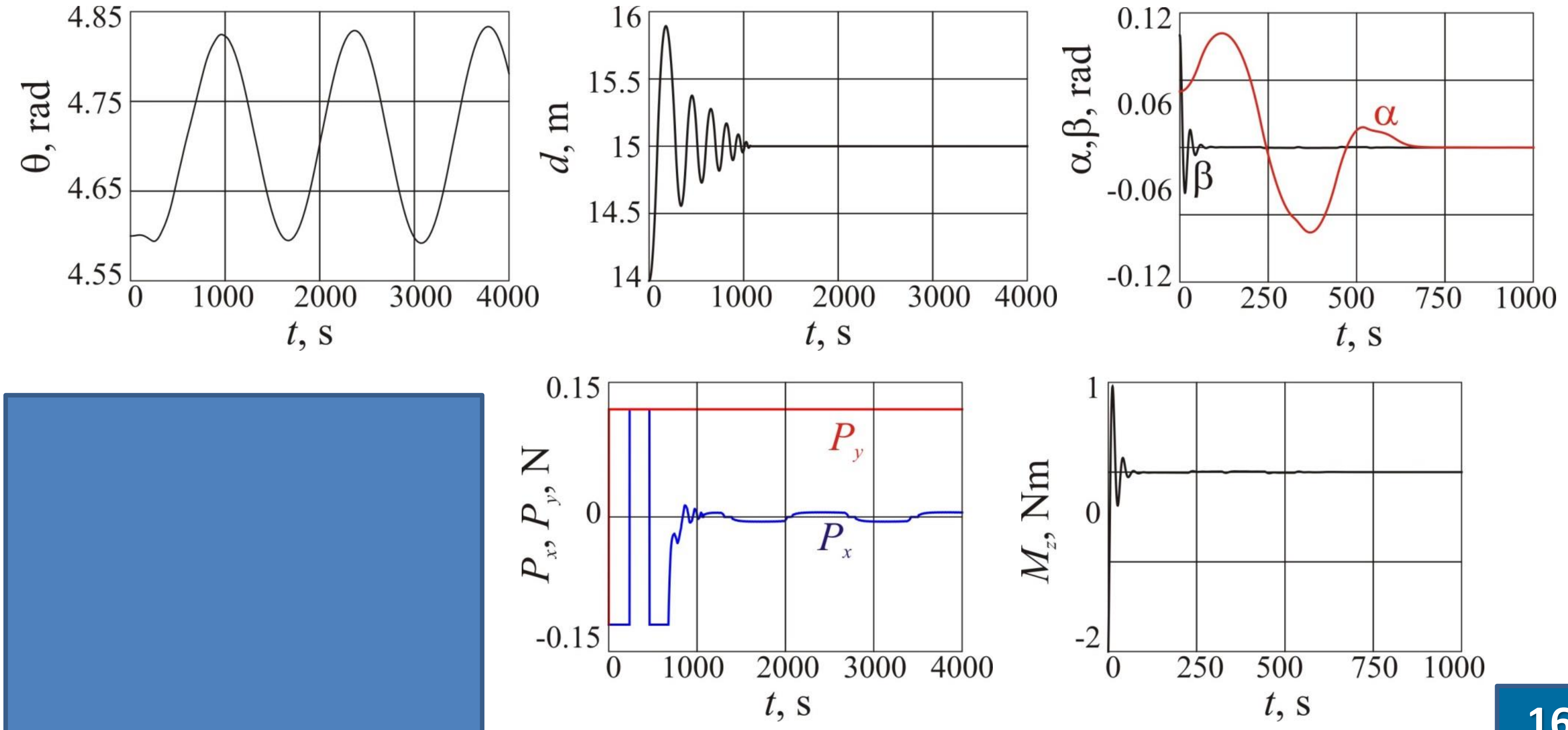


Space debris control

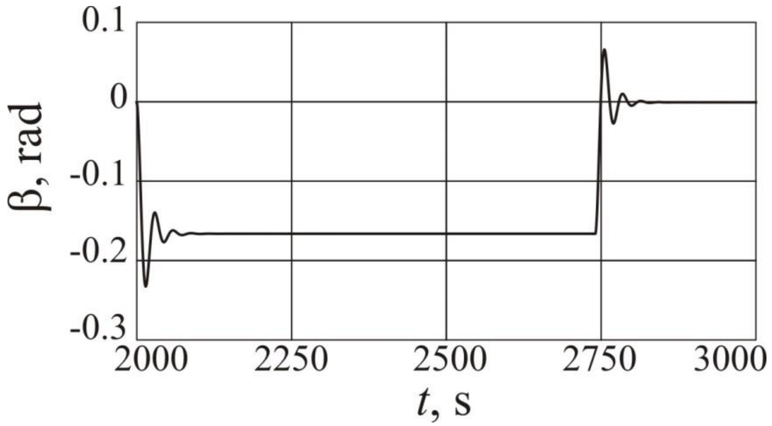


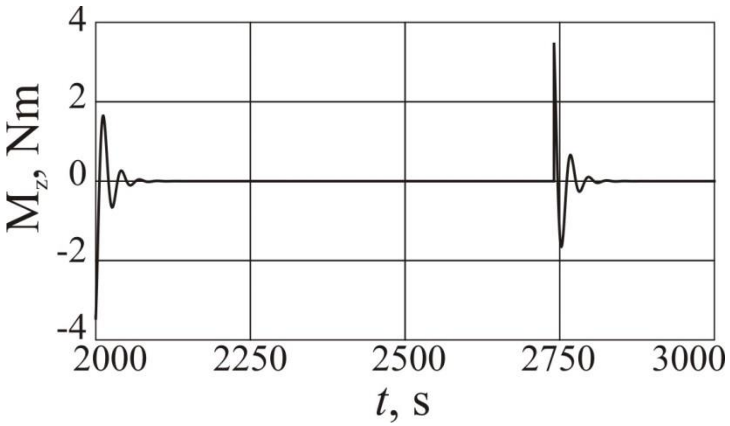
$$M_z = \begin{cases} (\beta_1 - \beta)k_{1\beta} - \dot{\beta}k_{2\beta}, & when \quad L_z(\theta, \beta_1)\dot{\theta}(E_* - E) > 0, \\ (\beta_2 - \beta)k_{1\beta} - \dot{\beta}k_{2\beta}, & when \quad L_z(\theta, \beta_2)\dot{\theta}(E_* - E) > 0, \\ -\beta k_{1\beta} - \dot{\beta}k_{2\beta} & otherwise \end{cases}$$

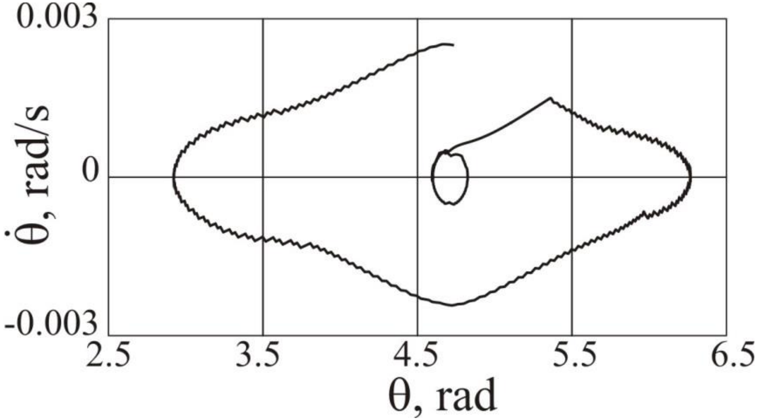
Results of numerical simulation



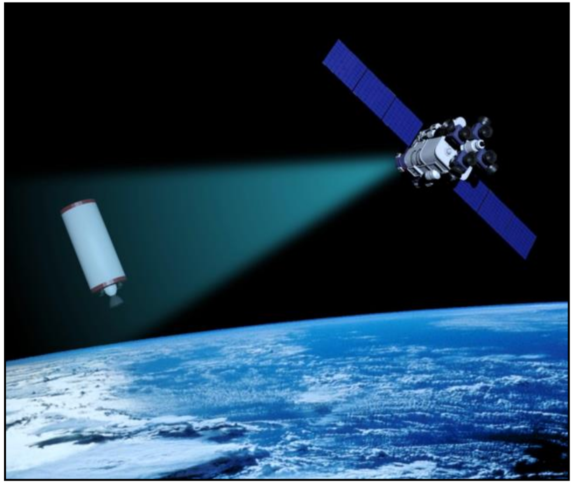
Results of numerical simulation

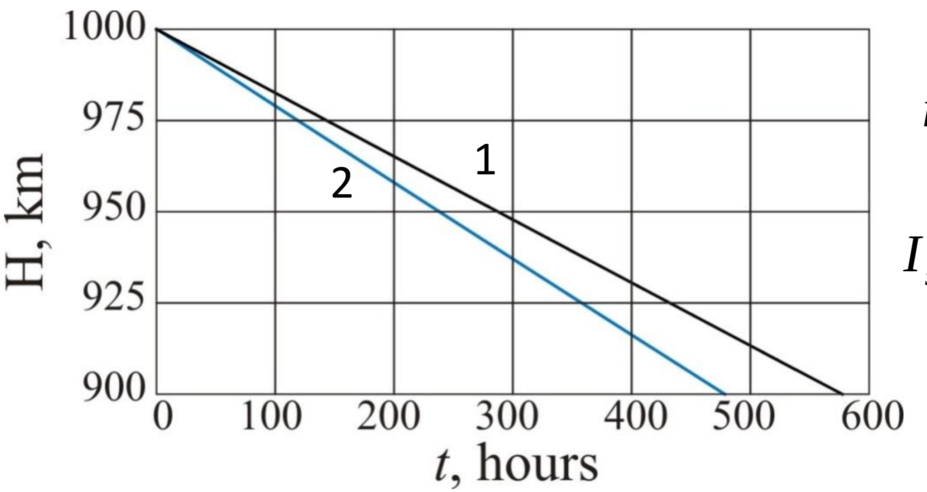






Results of numerical simulation





$$\dot{m} = \frac{T}{I_{sp} g_0}$$

$$I_{sp} = 2000s$$

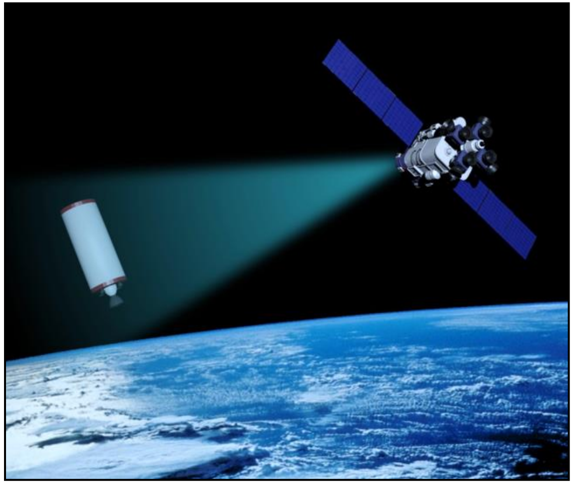
$$m_1 = 11.1648\text{kg}$$

$$m_2 = 9.3418\text{kg}$$

$$\Delta t = 98.76 \text{ hours}$$

$$\frac{m_1 - m_2}{m_2} = 0.1633$$

Conclusions and results



- The mathematical model was developed using the Lagrange formalism.
- The undisturbed oscillations of the cylindrical space debris were studies.
- A phase trajectory on which the average ion beam force is maximum in absolute value was determined.
- The control law of the spacecraft orientation engines, which ensures the transfer of the space debris object into a motion along the phase trajectory with maximum average ion beam drag force, was proposed.
- It was shown that the attitude motion of a spacecraft during transportation has a significant effect on the required fuel costs.

Thank you!

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